



Hybrid modal design of plates with distributed resonators for broadband vibration control

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Abstract

Reducing vibration can be very challenging in plate like structures with multiple modes triggered due to broadband loading and considerable constraints on additional mass. This study presents a hybrid approach to the placement and evaluation of distributed resonators attached on a fully clamped plate. The method combines finite element analysis based modal data with a reduced order analytical model. Instead of traditional parameter optimization, resonator locations are calculated by a methodical approach guided by weighted modal displacement maps. Three weighting approaches are applied such as equal weighting, effective modal mass weighting, and a displacement based weighting inspired by the physics inherent in the MIL-STD-810H standard. The final designs are evaluated using displacement frequency response functions. The results show how the chosen weighting clearly specifies the overall vibration suppression across the spectrum as well as the spatial distribution of resonators.

1. Introduction

Controlling mechanical vibrations in engineering is very important for the integrity of the system and especially for the functionality of aerospace and defense related applications [1,2]. Among passive control methods, Tuned Mass Damper (TMD) systems are commonly used because of their mechanical simplicity, low cost, and reliability [3-7]. The fundamental concept of the TMD, which was first introduced by Frahm [5], has been successfully applied to systems ranging from simple models with a single degree of freedom to complex multi degree systems in many engineering areas, especially in civil and mechanical engineering [6,7]. Integrating damping elements into TMD configurations has improved their robustness against mistuning and enabled efficient vibration reduction over a broader frequency range [8]. Nonetheless, a single TMD unit may demonstrate inadequate performance in more complex structures or systems due to the limited effective bandwidth when broadband excitation is considered [9,35]. A composite TMD (CTMD), distributed TMD (DTMD), and multiple TMD (MTMD) system has been suggested to overcome this issue [10,11] as an effective and practical solution. The DTMD systems utilize several resonators adjusted to frequencies that address various vibration modes of the host structure,

enhancing attenuation capabilities across a broader range [12,13]. The development and adjustment of these systems have been carried out through both traditional analytical methods, such as the Den Hartog approach, and sophisticated numerical optimization strategies, like genetic algorithms and simulated annealing [14-16]. In addition, several studies have investigated optimal MTMD configurations under harmonic excitation and the influence of manufacturing variations on control performance [33,37]. However, many of these approaches rely on iterative optimization procedures, which may not always be practical under strict mass and attachment constraints.

Recently, mechanical metamaterials have become popular for vibration control, especially the locally resonant designs introduced by Liu et al. [18]. In contrast to Bragg scattering that relies on periodicity similar to the wavelength, locally resonant metamaterials employ sub-wavelength resonators to produce low-frequency bandgaps [19,20]. These geometrically designed structures enable the effective reduction of vibrational energy within specific frequency ranges [17,36]. Recent research has shown the capability of these metamaterial/metastructures for isolating vibrations in beams [21], plates [22], and sandwich panels [23]. Additionally, developments in additive manufacturing have

made it easier to precisely fabricate resonator units with complicated lattice morphologies, like gyroids or Kelvin cells, greatly increasing the potential of metamaterial inspired vibration mitigation solutions [24-26]. For continuous structures such as beams and plates, the success of vibration control relies significantly on precise estimation of modal parameters and strategic placement of resonators [27,28]. Previous studies have shown that placement of resonators on antinodal regions, where vibration energy is concentrated, enhances energy transfer from the host structure to the absorbers [29,34]. However, most optimization strategies in the literature assume equal importance for all vibration modes or rely on uniform distributions [30]. In practical situations, such as the random vibration conditions outlined by standards like MIL-STD-810H, spectral weighting techniques that focus on low-frequency modes are important to reduce displacement responses [31,32].

Inspired by mechanical metamaterials and the principles of distributed tuned mass dampers, this study investigates the analytical performance of placing distributed resonators on a plate structure. A hybrid modal-domain modeling is employed, integrating modal data obtained from finite element analysis with an analytical structural modification method to obtain the vibration response. The suggested method is illustrated on a clamped rectangular plate, specifically a 250×250×2 mm steel plate used as a benchmark, equipped with four resonator units. The placement of resonators is determined by modal displacement maps that employ several types of weighting methodologies, including equal weighting, effective modal mass weighting, and a low-frequency weighting inspired by the MIL-STD-810H environment. Instead of using the standard directly as an excitation source or optimizing the parameters, the displacement dominance of the standard is used to guide a systematic placement and tuning process. Finally, vibration mitigation performance is assessed using displacement frequency response functions, with a specific attention to broadband RMS trends. Based on the theoretical framework of passive vibration control, the proposed methodology can be classified as a distributed tuned mass damper system for the continuous plate, matching the criteria set by Yang et al. [10]. In parallel to the current problem, the application of distributed resonators for localized resonance also categorizes the proposed study into the wider topic of mechanical metamaterials, as recently reviewed by Fayyaz et al. [17]. The main contribution of this study is to show that different modal weighting approaches can guide resonator placement in a systematic way, without the need for iterative optimization.

2. Hybrid modeling methodology

The proposed method combines the geometric accuracy of finite element analysis (FEA) with the computational effectiveness of analytical modal-domain modeling. The host structure, a rectangular steel plate, is modelled and analysed

with ANSYS Mechanical APDL (Release 2024 R1). The plate is simplified using four-node SHELL181 elements, which are suitable for thin plate vibration analysis. A constructed mesh of 2500 elements and 2601 nodes is used to precisely represent the first six flexural vibration modes. The boundary conditions are fully clamped across all four edges. Table 1 summarizes the host plate's geometric dimensions and material properties.

According to the placement approach, resonators are attached to specific nodes that have high modal participation. At each attachment point, the resonator is represented as a 2-DOF unit with two parallel branches, each having a lumped mass, spring, and damper. These branches are tuned separately to the two vibration modes having the highest local amplitude at that node. Fig. 1 illustrates the finite element mesh, clamping regions, and selected conceptual attachment placements.

Modal frequencies and corresponding mode shapes obtained from the FEA model are exported for every node and transferred to a MATLAB environment. This data is post-processed to ensure unit consistency and formatting for the subsequent modal-domain formulation. The first six flexural mode shapes considered in this study, along with their natural frequencies, are shown in Fig. 2. Second and third modes form a nearly identical-frequency (degenerate) pair arising from geometric symmetry, representing orthogonal mode shapes.

The dynamics of the system, comprising the host plate and R distributed resonator units, are formulated using modal superposition. Generalized mass, stiffness, and damping matrices are assembled in the modal domain, where each resonator branch is coupled to the plate through the local modal displacement vector at its attachment node. For each branch, the resonator stiffness is determined from the target tuning frequency as

$$k_r = m_r \omega_r^2 \quad (1)$$

where m_r is the lumped resonator mass and $\omega_r = 2\pi f_r$ is the selected tuning frequency. The steady-state response under harmonic excitation is obtained by solving the modal-domain FRF equation

$$(\mathbf{K}_{sys} - \omega^2 \mathbf{M}_{sys} + i\omega \mathbf{C}_{sys})\mathbf{Q} = \mathbf{F}_{modal} \quad (2)$$

Table 1. Geometric and material properties of the host plate

Property	Value
Material	Structural steel
Young's modulus, E	200 GPa
Poisson's ratio, ν	0.30
Density, ρ	7850 kg/m ³
Nominal plate dimensions	250×250×2 mm
Effective vibrating area	240×240×2 mm
Boundary condition	Fully clamped (5 mm per edge)

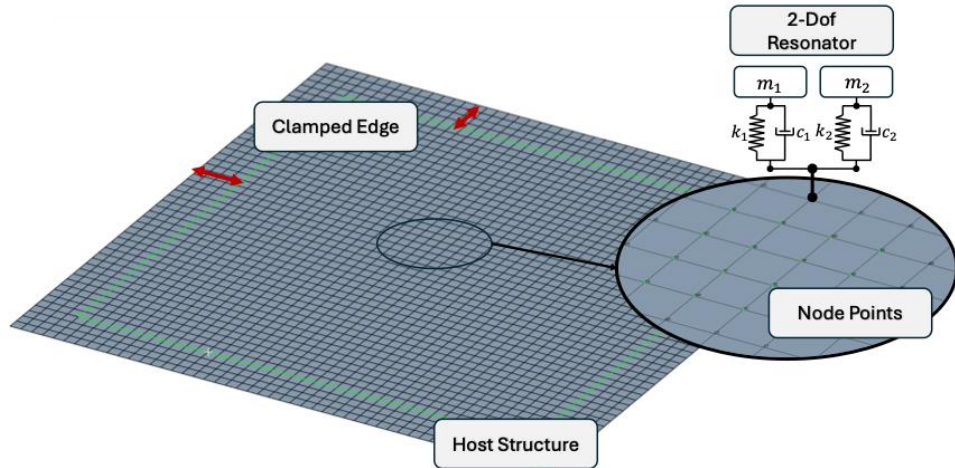


Fig. 1. Finite element model of the clamped plate and schematic of the resonator attachment

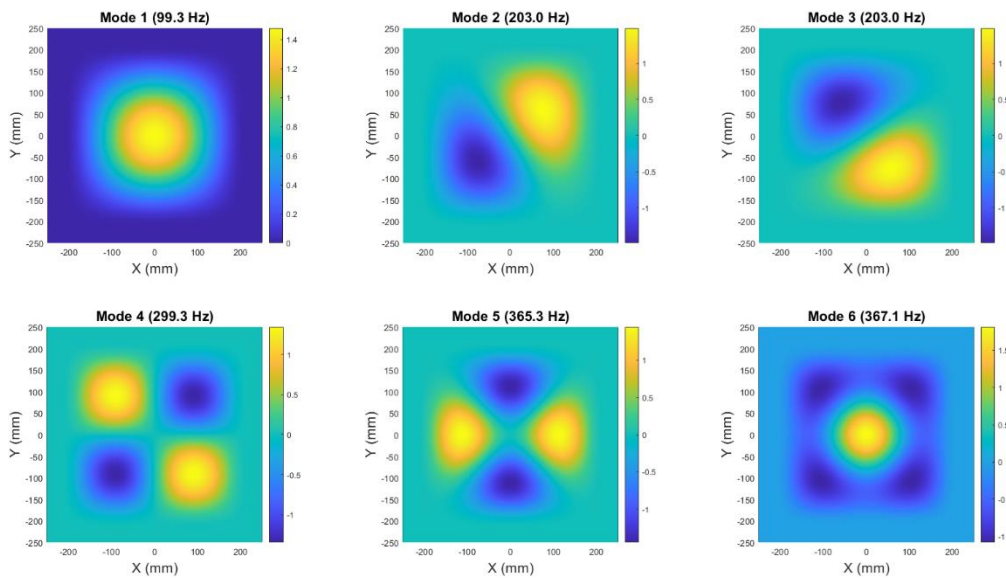


Fig. 2. First six flexural mode shapes of the clamped plate, with corresponding natural frequencies

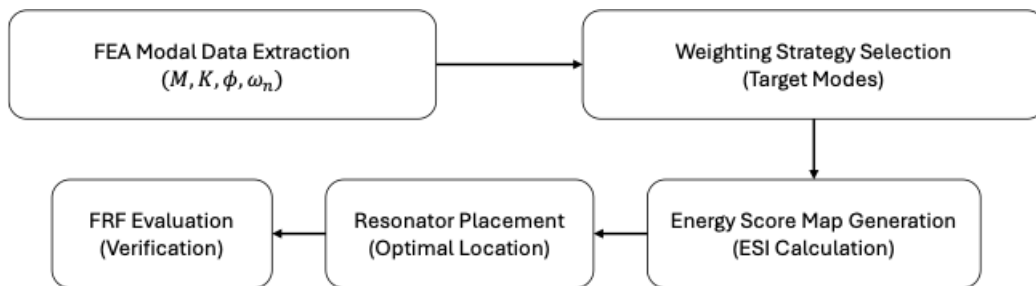


Fig. 3. Workflow of the resonator placement method

Table 2. Modal weighting approaches used for resonator placement

Approach	Weighting function	Physical considerations	Objective
Equal weighting	$w_k = 1$	Treats all vibration modes with equal importance	Balanced (all modes)
Effective modal mass	$w_k \propto M_{\text{eff},k}$	Prioritizes modes with high global mass participation	Global modes
MIL-STD-810H	$w_k \propto 1/f_k^2$	Targets displacement dominance under constant acceleration	Low-frequency modes

where \mathbf{Q} denotes the vector of modal coordinates. This governing equation represents a reduced-order, modal-domain formulation of the coupled plate-resonator system. The structural dynamics of the continuous plate are first projected onto a truncated modal basis derived from finite element analysis, after which the resonator branches are introduced through analytical structural modification. This hybrid formulation enables fast evaluation of vibration responses without the need for repeated finite element investigations, while maintaining the important spatial properties of the mode shapes by nodal coupling. The excitation force is applied off-center to disrupt geometric symmetry and stimulate both symmetric and antisymmetric mode shapes, including practically degenerate pairs, at the same time. As a result, the analyzed frequency response functions provide a more realistic broadband excitation scenario, eliminating the artificial suppression of modal contributions that might occur with symmetric loading. For simplicity and consistency, the host plate and resonators are considered to have constant viscous damping ratios. The plate modal damping ratio is set to $\zeta_p = 0.01$, while the resonator damping ratio is fixed at $\zeta_r = 0.05$ for all branches. The viscous damping coefficient of each resonator branch is calculated as

$$c_r = 2\zeta_r m_r \omega_r \quad (3)$$

where m_r is the resonator mass and ω_r is the tuning frequency. In this work, damping is not considered a design variable. It is kept constant in order to focus only on the effect of the proposed modal-weighted placement strategy. In this study, each attachment location is modeled as a 2-DOF resonator unit consisting of two parallel branches. This configuration is selected to allow simultaneous tuning to two dominant vibration modes that exhibit high local modal amplitude at the same node. A 1-DOF absorber would only target one resonance frequency, whereas the proposed two branch configuration increases local modal interaction without increasing the number of attachment locations. This approach provides multi modal mitigation while maintaining a limited number of resonator units and a controlled mass penalty.

3. Modal domain design strategy

In this study, resonator placement is determined by objective-based weighting methods to better represent vibration mitigation requirements under specific operating conditions. Rather than optimizing resonator parameters in the traditional manner, this study aims to identify spatial regions of the host plate that are most sensitive to vibration under realistic operating and fundamental dynamic conditions. Within this framework, the same placement methodology is employed for all cases, while only the modal weighting scheme is varied. Accordingly, three distinct modal weighting strategies w_k applied to modes $K = 1, \dots, N$ are evaluated, as summarized

in Table 2. It is important to note that this study does not attempt to address a formal parameter optimization problem. Instead, a modal-based placement approach is used. The resonator locations and tuning frequencies are directly determined from the weighted modal characteristics of the host structure.

The equal weighting method serves as a baseline, treating all resonance modes equally important. The effective mass approach improves this assumption by applying finite element calculated modal effective mass ratios, selecting modes that make a major contribution to the global structural response. The weighting parameters are dependent on the solver output's out-of-plane (Z-direction) effective modal mass participation ratios, which ensure that modes with significant transverse response characteristics are highlighted. The third approach is inspired by the physical characteristics of the MIL-STD-810H random vibration environment (Jet Aircraft category) [32]. Under broadband excitation with almost constant acceleration content, displacement scales with $1/f^2$, leading to low-frequency modes dominating the response. Modal weights are calculated as $w_k \propto 1/f_k^2$ to emphasize lower frequency modal content. It should be noted that the MIL-STD-810H profile is not employed as a direct input excitation spectrum in this case; rather, its physical implications for displacement dominance are used to guide the weighting method.

Using the proposed weighting method, an energy score map $S(x, y)$ is generated that demonstrates the spatial distribution of vibration intensity throughout the plate surface. The score map is a weighted superposition of the absolute modal displacements:

$$S(x, y) = \sum_{k=1}^N w_k |\phi_k(x, y)| \quad (4)$$

Highly ranked locations represent areas where multiple weighted modes have an important effect on the local vibration response. These areas are indicated as potential attachment places for resonator units. The previous section explains how a 2-DOF resonator unit is tuned to the two dominant modes for each node. Exclusion zones are defined around critical locations such as the excitation source and attachment placements to ensure physical feasibility and prevent mechanical interference with supporting components. In addition, a strict mass constraint is imposed to maintain lightweight design principles. The resonator system's relative additional mass is quantified using the mass penalty ratio, which is defined as

$$\mu_{pen} = \frac{\sum m_{res}}{M_{plate}} \quad (5)$$

where $\sum m_{res}$ and M_{plate} represent the total mass of the attached resonator units and the mass of the host plate, respectively. In this study, the mass penalty is set to

approximately $\mu_{pen} \approx 0.05$ in order to balance vibration mitigation performance and additional mass.

4. Results and discussions

In this study, the total number of resonator units is limited to four. This number is selected to maintain a clear comparison between the different modal weighting approaches while keeping the total added mass within a practical range. Increasing the number of resonators would generally improve broadband vibration suppression. However, the objective of this work is not to maximize performance through high-density absorber distribution, but to evaluate the influence of the proposed placement strategy under a fixed mass constraint. Therefore, the number of resonators is kept constant in all configurations to ensure a fair comparison. Numerical analyses are carried out on both the bare plate and three configured plates with distributed resonators. The configurations correspond to different modal weighting methodologies, including equal weighting, effective modal mass weighting, and spectral weighting using the MIL-STD-810H (Jet Aircraft) random profile. In all cases, four resonator units are used, and each unit consists of two 5 g masses, leading to a total added mass of 0.040 kg and a mass penalty of approximately 4.1% relative to the host plate. The selection of modal weighting approach is demonstrated to have a considerable impact on resonator location. As shown in Fig. 4, the equal weighting approach provides a scattered placement pattern. The resonators are distributed over the plate surface to address both low and high frequency vibration modes. In contrast, the effective mass approach focuses resonators on antinodal regions associated with globally participating modes, while ignoring localized regions with high modal amplitude. The MIL-STD approach focuses on low-frequency modes with $1/f^2$ displacement scaling, leading to a high concentration of resonators around the antinodes of the first and second flexural modes. This placement pattern is consistent with the weighting approach described in Section 3 and highlights the importance of low-frequency modes in displacement-induced vibration responses.

Following the placement process, the tuning frequencies and corresponding stiffness values of the 2-DOF resonator units are calculated based on the locally dominant modal characteristics identified by each approach. Table 3 summarizes the tuning parameters of the resonator units for each configuration. For each weighting approach, the tuning frequencies are selected from the dominant modal frequencies at the chosen attachment locations. In the present case, this resulted in identical tuning parameters for all four resonators within a given strategy, as shown in Table 3. However, the tuning differs between the proposed weighting methods due to the different modal priorities. In this study, the tuning

parameters are the same within each strategy, but the resonator locations differ considerably between the effective modal mass and MIL-STD-810H designs. As shown in Fig. 4, the effective mass approach produces a more central placement, while the MIL-STD-810H method concentrates the resonators near the antinodes of the low frequency modes. This observation demonstrates that the weighting method affects resonator placement rather than tuning, and spatial dispersion is critical in defining the final vibration response.

Furthermore, the vibration mitigation performance is evaluated using displacement frequency response functions. Fig. 5 compares the bare plate and three plate configurations with distributed resonators, and Table 4 summarizes the global performance metrics of the proposed approaches using RMS evaluation across all frequency bands. The equal weighting configuration provides uniform reduction across the frequency band, reflecting the balanced handling of all vibration modes. However, this method provides limited suppression at the fundamental resonance. This is because low frequency dominant modes are not emphasized. The effective mass configuration has better reduction at specific resonances associated with modes that contribute significantly to the global response. This pattern is compatible with the presence of resonators near areas of significant modal mass participation. However, the ensuing vibration reduction is not consistent across the spectrum, with a reduced effectiveness at higher frequencies. The configuration based on MIL-STD-810H has the most considerable reduction at the fundamental resonance, as shown in Fig. 5. The $1/f^2$ scaling's displacement-based weighting stimulates low-frequency modes under constant acceleration excitation, resulting in this response. Table 4 shows that this configuration achieves a significant broadband RMS reduction over all of the frequency band (50-450 Hz) compared to other configurations.

Table 3. Tuning parameters (frequency in Hz and stiffness in N/m) of the 2-DOF resonators for each weighting strategy

DOF	Equal weighting		Effective modal mass		MIL-STD-810H	
	f_r	k_r	f_r	k_r	f_r	k_r
1 st	202.97	8131.9	367.08	26598.1	99.30	1946.4
2 nd	299.26	17677.8	99.30	1946.4	367.08	26598.1

Table 4. Broadband vibration reduction compared to the bare plate

	Equal weighting	Effective modal mass	MIL-STD-810H
Broadband RMS reduction (dB)	1.59	2.83	3.02
Overall vibration reduction (%)	16.70	27.83	29.38

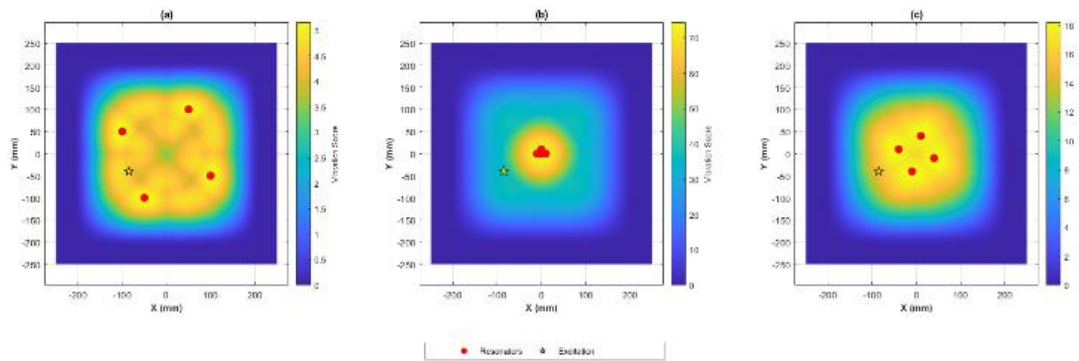


Fig. 4. Resonator placement for different modal weighting approaches: (a) equal weighting, (b) effective modal mass weighting, and (c) spectral weighting based on MIL-STD-810H

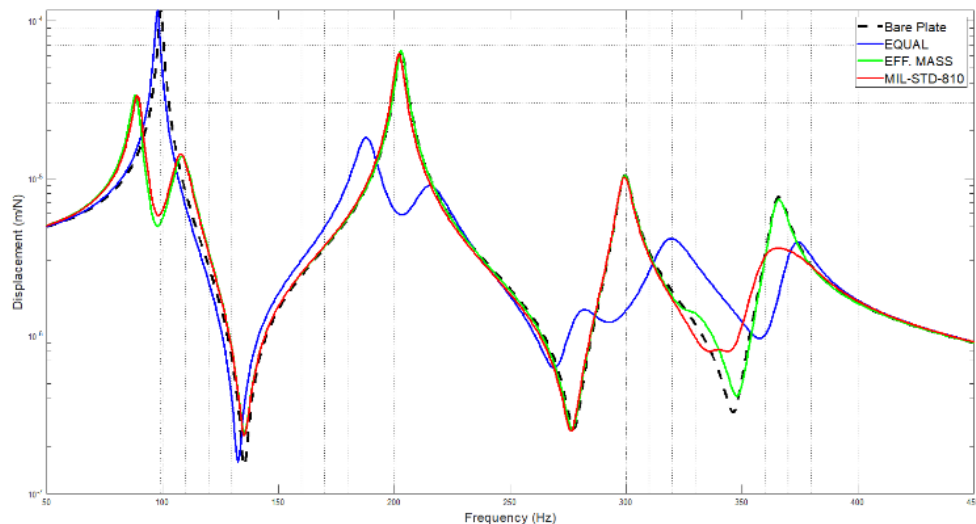


Fig. 5. Comparison of displacement FRFs for the bare plate and the resonator-enhanced configurations

It is also worth mentioning that this study actually limits the number of resonators to four. While it is well-known that larger number of smaller units increases broadband suppression and durability against detuning, we maintained the number of units identical to isolate the specific impact of the weighting methods. This approach ensures that the observed performance differences are brought about only by the placement logic, rather than the cumulative effects of a high density resonator distribution.

Overall, the comparison results show that the selected modal weighting approach has an important effect on both the spatial location of resonators and the vibration response. Rather than proposing a single optimal solution, the study shows how different weighting approaches influence vibration mitigation performance under various operating conditions. The observed improvements agree with the general behavior reported for distributed tuned mass damper systems, where resonator placement and tuning greatly influence modal energy transfer [10]. The results confirm recent discoveries in the mechanical metamaterials literature, highlighting the usefulness of locally resonant attachments for low frequency vibration mitigation [17]. It should be noted that the primary objective of this study is not to compare single versus multiple TMD configurations, but rather to

investigate how different modal weighting strategies influence the placement of distributed resonators under broadband excitation. Nevertheless, a single TMD tuned to the fundamental frequency would reduce vibration only near that resonance, while the other modes would remain largely unaffected. In contrast, the proposed distributed framework targets multiple dominant modes and enables simultaneous vibration reduction within the same total mass budget. This multimodal capability is the fundamental motivation for adopting a distributed configuration in the present study. Therefore, the bare plate is selected as the reference case to isolate the effect of the proposed placement strategy, rather than to benchmark against a single TMD system.

5. Conclusions

This paper presents a hybrid modal domain approach for placement and evaluating distributed resonators on a four edge clamped plate under broadband excitation. By combining modal data from finite element analysis with an analytical structural modification method, vibration responses can be evaluated efficiently without iterative optimization or repeated finite element analyses. Rather than identifying a single best design, the study compares different modal weighting approaches and examines their effects on

resonator placement, tuning, and vibration response. The results show that the specified weighting approach clearly affects both the spatial distribution of resonators and the frequency dependent damping behavior. Weighting approaches that prioritize low frequency modal content provide better reduction at the fundamental resonance, whereas balanced or mass based approaches offer broader but less targeted reductions. Considerable vibration mitigation is achieved in all configurations with minimal additional mass, supporting the value of distributed resonator principles for lightweight structural applications. Overall, the findings emphasize the significance of matching resonator placement strategies to the specific properties of the excitation, resulting in a clear and physically understandable framework to develop resonator enhanced plate structures. Thus, the proposed framework combines traditional distributed TMD ideas with design philosophies influenced by locally resonant metamaterials, all while conforming to practical engineering limitations. Future work will center on experimental validation and expanding the method to different excitation settings.

Declarations

Conflict of interests

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Data availability statement

The data presented in this study are available on request from the corresponding author.

Use of generative AI and AI-assisted technologies

The author(s) confirm the author(s) did not use any AI tools in the preparation of this work/research/study.

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