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RESEARCH ARTICLE

Analytical, numerical, and experimental determination of dynamic characteristics of prestressed concrete girders

Barbaros Atmaca, Şevket Ateş, Murat Günaydın

Karadeniz Technical University, Department of Civil Engineering, Trabzon, Türkiye

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Abstract

To determine the dynamic characteristics of bridges with prestressed girders, their natural frequencies and mode shapes used to construct the superstructure must be known. However, there is no agreement among scientists on how prestress force affects the dynamic characteristics of a precast prestressed girder. The purpose of this paper is to obtain the dynamic characteristics of prestressed concrete girders through analytically, numerically, and operational modal testing. For this purpose, one of a typical precast I-girder with 1.2 m height and 27.45 m effective span length is selected as a numerical application. The three-dimensional (3D) finite element model (FEM) of the girder is modeled by SAP2000. Experimental measurements of the girder were conducted at the construction site by the operational modal testing method. For experimental measurements, ten uniaxial seismic accelerometers were mounted at the top flange of the PSC girder in the *x*- and *z*-direction. The vibrations that occur from the movement of trucks and cranes at the construction site and the impact of the hammer were measured by these accelerometers as acceleration. The measured signal was collected at the data bank and then sent to the computer equipped with Operational Modal Analysis software which used Enhanced Frequency Domain Decomposition and Stochastic Subspace Identification techniques. The dynamic behaviors of girder were derived from analysis performed by this software. At the end of the study, the dynamic characteristic was obtained by the analytical prediction, numerical and experimental were compared with each other. It is seen that natural frequencies and mode shapes obtained from theoretical prediction, numerical and operational modal testing are not too far apart.

1. Introduction

The increasing world population triggers an increase in the demand for transportation. Bridges are one of the most important components of transportation and there are different types of bridges to meet different needs. Prestressed concrete (PSC) I-girder bridges are preferred to construct extensively in medium span (15 m to 40 m) highway bridges. The US national bridge inventory (NBI) and Turkish General Directory of Highways data show that the PSC bridges cover a significant percentage of the existing bridges. Moravcik [1], Bujnakova, and Strieska [2] stated that a significant portion of the highway bridges in Slovakia was produced with prestressed concrete technology and that about 200 km of the new bridge, according to recent plans, would be made using PSC technology. It is also clear that the construction of PSC bridges will continue to

Correspondence Barbaros Atmaca atmaca@ktu.edu.tr

increase when the increasing transportation needs of the communities and superior properties of PSC are considered. One of the construction views of the PSC I-girder bridges is given in Fig.1.

The natural frequency (NF) and mode shape (MS) are important modal parameters of engineering structures. To determine these parameters of bridges built with prestressed concrete (PSC) or reinforced (RC) beams, the dynamic properties of such beams should be known to obtain the current state of the bridge for damage detection. Dynamic characteristics are also an important reference for earthquake-resistance design so it is very important to obtain the theoretical modal parameters of PSC girders correctly in risky areas for earthquakes [4]. PSC girders are subjected to prestressing force (PF) different from RC beams. Dynamic behavior of simply supported girder is existing in literature but the effect of PF on the dynamic behavior of PSC girder has been a debated topic and still, there is no agreement about how PF affects the modal parameters of PSC girder. The scientific community reported contradictory results on this topic. Some reports state that PF affects the dynamic behavior of PSC girder. Tse et al. [5], Saiidi et al. [6], and Chan and Yung [7] emphasized that compression softening, is a gradual reduction of mechanical resistance due to a continuous increment of deformation forced upon a concrete, affects the dynamic characteristic of PSC beam and the higher level of PF causes to reduce the NFs of the beam. Grace and Rose [8] point out that the modal parameters especially NFs of girder are affected by the level of PF and location of prestressing tendon at the bottom flange of the girder. However modal shapes are less affected by such parameters. Miyamoto et al. [9] struggled to identify NFs and MSs of a prestressed beam that stiffened with external tendons. They have concluded that NFs show decreasing tendency when the magnitude of PF boost. The NF of a simply supported axially compressed beam is

$$w_n = \frac{n\pi}{L_{girder}} \sqrt{\frac{1}{m_{girder}} \left[EI \left(\frac{n\pi}{L_{girder}} \right)^2 - N \right]} \tag{1}$$

where EI is the flexural rigidity of the girder, L_{girder} is the length of the girder, m_{girder} is the mass per unit length of the girder, n is the mode number and N is the axial compressive force [10]. Eqs. (1) demonstrates that when the value of the PF rises, the NF of the axially compressed simple supported beam tends to decrease. As seen from many reports, the PF affects the NFs of the PSC girder, whereas some reports state that the PF has no effect. Deak [11] put forward that the NFs were not affected by the level of the prestress force. However, this claim has not been proved by any comprehensive work. Hamed and Frostig [12] examined how the NFs of prestressed beams which have bonded or unbonded tendons were affected by applied prestress force. They were suggested a nonlinear analytical formulation to obtain the NF of the PSC beam. NFs of bonded or unbounded PSC beams were obtained from this suggested formulation. The results show that the NFs of selected beams were not affected by the level of applied prestress force. Pavic et al. [13] reached analogous results that the prestress forces do not have a significant impact on the dynamic properties. Only parameters such as mass, stiffness, and damping properties were affected by the NFs and MSs of the PSC girder. Noble et al. [14] conducted an experimental study to obtain the effect of post-tensioning force level on the post-tensioned concrete beam. For this purpose, they worked on various levels of PF with zero eccentricity. The result of this experimental study shows that magnitude of PF is not affected the NF. Fengge and Rong [15] emphasized that the NFs of unbounded PSC beams are not affected by the prestress force. It is seen that the references mentioned above, studies on the impact of PF on NFs of PSC girders are debated topics.

This paper aims to determine the dynamic characteristics of PSC girder through the analytical method suggested by [10], numerically by FEM, and operational modal testing. For this purpose, one of the typical precast I-girder is selected as an example. The 3D FEM of the girder is created with SAP2000 [16]. The experimental measurements of the girder were conducted at the construction site by operational modal testing. For experimental measurements, ten uni-axial seismic accelerometers were mounted at the top flange of the PSC girder in *x*- and *z*-direction. The vibrations that occur from the movement of trucks and cranes at the construction site and also the impact of the hammer were measured by these accelerometers as acceleration. The measured signal was collected at the data bank and then sent to the computer equipped with Operational Modal Analysis (OMA) [17] software which used Enhanced Frequency Domain Decomposition (EFDD) and Stochastic Subspace Identification (SSI) techniques. The dynamic behaviors of girders were derived from the analysis performed by this software.

2. Modal parameter estimation techniques

The operational modal analysis (OMA) is evaluated in this paper as a technique for the identification of modal parameters. Many modal parameter identification techniques developed by researchers for different uses. In this study, the enhanced frequency domain decomposition (EFDD) method in the frequency domain, and the more advanced stochastic subspace identification (SSI) method in the time domain among these techniques are implemented to extract the modal parameters.

2.1. Enhanced frequency domain decomposition (EFDD) method

The EFDD method is primarily based on the fact that the frequency response function goes through an extreme around the natural frequencies. In the context of ambient vibration measurements, the frequency response function is replaced by the auto spectra of the output-only data. To include the measurement channels of all setups, the average normalized power spectral densities (ANPSDs) are used. In such a way, the identified natural frequencies are simply obtained from the observation of the peaks on the graphs of ANPSDs.

The relationship between the input $x(t)$ and the output $y(t)$ can be written [18] and [19]:

$$\left[G_{\mathbf{yy}}(\omega)\right]=\left[H(\omega)\right]^{*}\left[G_{\mathbf{xx}}(\omega)\right]\left[H(\omega)\right]^{T} \tag{2}$$

where $G_{\mathbf{xx}}$ is the Power Spectral Density (PSD) matrix of the input, $G_{\mathbf{yy}}$ is the PSD matrix of the output, H is the Frequency Response Function (FRF) matrix, and * and T denote complex conjugate and transpose respectively. After some mathematical manipulations the output PSD can be reduced to a pole/residue form as follows [18]:

$$\left[G_{yy}(\omega)\right]=\sum_{k=1}^{m}\left(\frac{[A_k]}{j\omega-\lambda_k}+\frac{[A_k]^{*}}{j\omega-\lambda_k^{*}}+\frac{[B_k]}{-j\omega-\lambda_k}+\frac{[B_k]^{*}}{-j\omega-\lambda_k^{*}}\right) \tag{3}$$

Fig. 1. Cross-section of the investigated girder [3]

where A_k is the kth residue matrix of the output PSD. The response spectral density matrix can be written in the following final form considering a lightly damped system [18]:

$$\left[G_{yy}(\omega)\right] = \sum_{k=Sub(\omega)} \left(\frac{d_k \psi_k \psi_k^T}{j\omega - \lambda_k} + \frac{d_k^* \psi_k^* \psi_k^T}{j\omega - \lambda_k^*} \right) \tag{4}$$

where d_k is a scalar constant and ψ_k is the kth mode shape vector. Thus, performing the singular value decomposition of the output PSD matrix known at discrete frequencies $\omega - \omega_i$ one obtains [18]:

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H \tag{5}$$

where the matrix U_i is a unitary matrix holding the singular vector u_{ij} and S_i is a diagonal matrix holding the scalar singular values s_{ij}. The superscript H denotes complex conjugate and transpose. Near a peak corresponding to the kth mode in the spectrum, only the kth mode is dominant, and the PSD matrix approximates a rank-one matrix as [18]:

$$\hat{G}_{yy}(j\omega_i) = s_i u_{i1} u_{i1}^H \quad , \qquad \omega_i \to \omega_k \tag{6}$$

The first singular vector at the rth resonance is an estimate of the rth mode shape [18]:

$$\hat{\phi}_r = u_{r1} \tag{7}$$

2.2. Stochastic subspace identification (SSI) method

The stochastic subspace identification technique is a time-domain method that directly works with time data, without the need to convert them to correlations or spectra. The stochastic subspace identification algorithm identifies the state-space matrices based on the measurements by using robust numerical techniques. Once the mathematical description of the structure (the state-space model) is found, it is straightforward to determine the modal parameters. The theoretical background is given in by Van Overschee and De Moor [20], as well as Peeters [21]. The model of vibration structures can be defined by a set of linear, constant-coefficient, and second-order differential equations [21]:

$$M\ddot{U}(t) + C_*\dot{U}(t) + KU(t) = F(t) = B_* u(t) \tag{8}$$

where M, C_*, K are the mass, damping, and stiffness matrices, $F(t)$ is the excitation force, and $U(t)$ is the displacement vector at continuous time t. Observe that the force vector $F(t)$ is factorized into a matrix B_* describing the inputs in space and a vector $u(t)$. Although Eq. (8) represents quite closely the true behavior of a vibrating structure, it is not directly used in SSI methods. So, the equation of dynamic equilibrium (8) will be converted to a more suitable form: the discrete-time stochastic state-space model [21]. The state-space model originates from control theory, but it also appears in mechanical/civil engineering to compute the modal parameters of a dynamic structure with a general viscous damping model [22]. With the following definitions,

$$x(t) = \begin{pmatrix} U(t) \\ \dot{U}(t) \end{pmatrix}, \quad A_c = \begin{pmatrix} 0 & I_{n_2} \\ -M^{-1}K & -M^{-1}C_* \end{pmatrix}, \quad B_c = \begin{pmatrix} 0 \\ M^{-1}B_* \end{pmatrix} \tag{9}$$

Eq. (9) can be transformed into the state equation

$$\dot{x}(t) = A_c x(t) + B_c u(t) \tag{10}$$

where A_c is the state matrix, B_c is the input matrix, and $x(t)$ is the state vector. The number of elements of the state-space vector is the number of independent variables needed to describe the state of a system. If it is assumed that the measurements are evaluated at only one sensor location and that these sensors can be accelerometers, velocity, or displacement transducers, the observation equation is [21]:

$$y(t) = C_a \ddot{U}(t) + C_v \dot{U}(t) + C_d U(t) \tag{11}$$

where $y(t)$ are the outputs, and C_a, C_v, C_d are the output matrices for displacement, velocity, and acceleration, respectively. With the following definitions [21]

$$\left.\begin{aligned} C &= \left[C_d - C_a M^{-1} K \quad C_v - C_a M^{-1} C_* \right] \\ D &= C_a M^{-1} B_* \end{aligned}\right\} \tag{12}$$

Eq. (11) can be transformed into:

$$y(t) = Cx(t) + Du(t) \tag{13}$$

where C is the output matrix and D is the direct transmission matrix. Eqs (10) and (13) constitute a continuous-time deterministic state-space model. Continuous-time means that the expressions can be evaluated at each time instant $t \in \mathbb{R}$ and deterministic means that the input-output quantities $u(t)$, $y(t)$ can be measured exactly. Of course, this is not realistic since measurements are available at discrete time instants $k\Delta t$, $k \in \mathbb{N}$ with Δt, the sample time and noise are always influencing the data. After sampling, the state-space model looks like this:

$$\left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}\right\} \tag{14}$$

where $x_k = x(k\Delta t)$ is the discrete-time state vector, $A = \exp(A_c \Delta t)$ is the discrete state matrix, and $B = \left[A - I \right] A_c^{-1} B_c$ is the discrete input matrix. If A_c is not invertible, another expression holds B. The stochastic components (noise) are included, and we obtain the following discrete-time combined deterministic-stochastic state-space model:

$$\left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned}\right\} \tag{15}$$

where w_k is the process noise due to disturbances and modeling inaccuracies and v_k is the measurement noise due to sensor inaccuracy. They are both immeasurable vector signals, but we assume that they are zero-mean, white, and with covariance matrices [21]

$$E\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix} \begin{pmatrix} w_q^T & v_q^T \end{pmatrix} \right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{pq} \tag{16}$$

where E is the expected value operator and δ_{pq} is the Kronecker delta.

The vibration information that is available in structural health monitoring is usually the responses of a structure excited by the operational inputs that are some immeasurable inputs. Due to the lack of input information, it is impossible to distinguish deterministic input u_k from the noise terms w_k , v_k in Eq. (15). If the deterministic input term u_k is modeled by the noise terms w_k , v_k the discrete-time purely stochastic state-space model of a vibration structure is obtained:

$$\left.\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned}\right\} \tag{17}$$

Eq. (17) constitutes the basis for the time-domain system identification through operational vibration measurements. The SSI method identifies the state-space matrices based on the output-only measurements and by using robust numerical techniques.

3. Prestressed concrete girder models

In this paper, a simply supported prestressed I-girder with 120 cm height and 27.45 m effective span length is selected as an application. A typical appearance and the dimensions of the cross-section are given in Fig. 2. The ultimate strength of concrete (f_c) is taken as 40 MPa (based on cylinders). The low-relaxation Grade270 prestressing strand (characteristic tensile strength f_u of 1860 MPa) 15 mm (0.6 in.) in diameter is selected as a strand type. Strands layout along the girder length is assumed as linear. The distance between strands is 6 cm. The modulus of elasticity, passion ratio, and density of concrete and strand is given in Table 1.

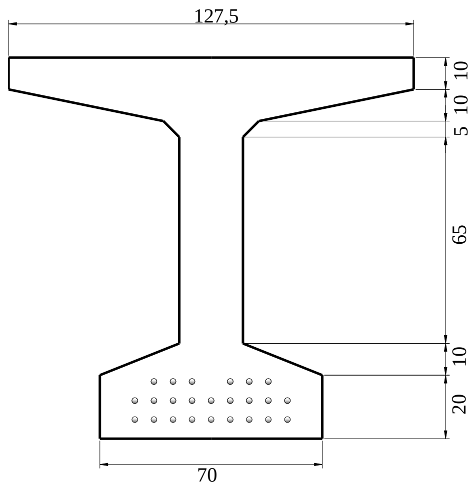

Fig. 2. Cross-section of the investigated girder

Table 1. Material properties of selected girder

Material	Density (kg/m^3)	Modulus of elasticity (MPa)	Poisson’s ratio
Concrete	2440	36000	0.2
Strand	7850	201600	0.3

4. Operational modal testing of the PSC girder

Operational Modal Analyses (OMA) [17] were performed to identify the exact dynamic characteristic of structures with the help of response vibrations. These response vibrations are acquired from structures then recorded at the data bank then sent for the processing phase with the help of PULSE [23] and OMA (2006) software which are operated EFDD and SSI techniques. To obtain the modal parameters, NFs and MSs, of PSC girder the operational modal testing was performed (Fig. 3). The uniaxial seismic accelerometers, which have a 1-1500 Hz frequency range and 10 V/g sensitivity were used to measure the response signal. To measure the exact structural response, enough points are determined on the top flange of the girder, and a total of ten accelerometers were mounted at the top flange of the PSC girder in *x*- and *z*-direction. The location of the mounting point of accelerometers on the PSC girder is shown in Fig. 4 schematically. The measurements were continued for 20 min and the response vibrations occurred from construction site effects such as truck and crane loads. In addition to these loads, an impact hammer was used to vibrate the girder to provide that all modes of the girder were obtained easily. Response signals that occurred from the ambient vibrations were measured by accelerometers. Then, these signals were recorded and sent to a computer equipped with OMA software. This software was analyzed the collected signal with the help of EFDD and SSI techniques. At the end of the analysis, exact dynamic characteristics such as the power spectral densities matrices, the stabilization diagrams, MSs, and NFs of the PSC girder were obtained. The power spectral densities matrices of the PSC girder obtained from the EFDD and SSI techniques are given in Figs. 5 and 6. The stabilization diagrams from the SSI technique are also given in Fig. 7.

NFs of the first four modes of PSC girder obtained from EFDD and SSI techniques are given in Table 2. Since the MSs obtained from both EFDD and SSI methods are very close to each other, only MSs of EFDD are given in Fig. 8.

Fig. 3. Operational model testing of the PSC girder

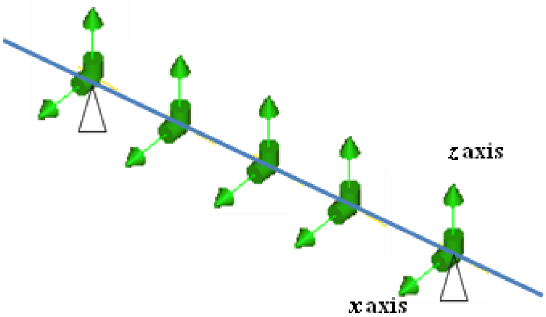

Fig. 4. The location of mounted point of accelerometers on PSC girder

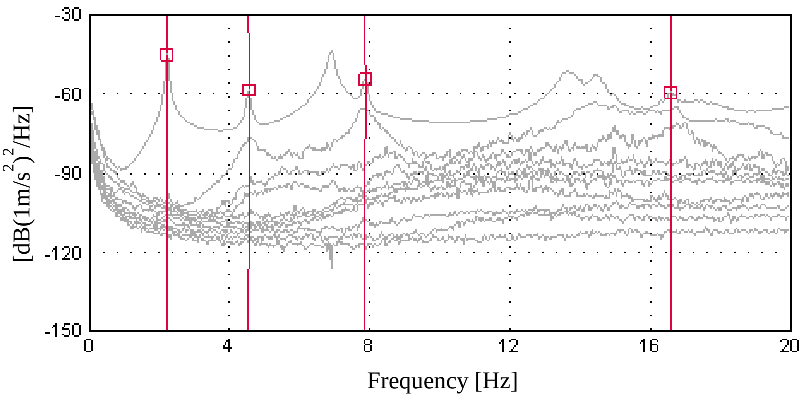

Fig. 5. The power spectral density function obtained from the EFDD technique

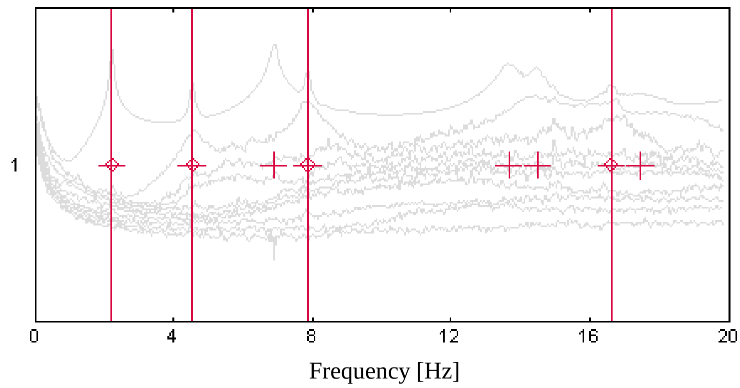

Fig. 6. The power spectral density function obtained from the SSI technique

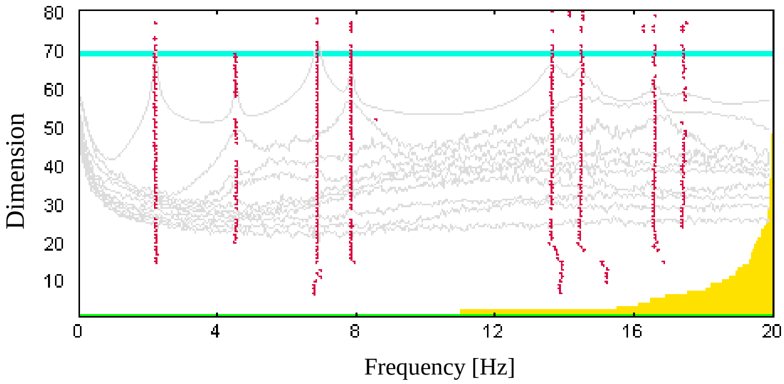

Fig. 7. The stabilization diagram of the SSI technique

Table 2. Natural frequencies of the first four modes of girder obtained from EFDD and SSI techniques

Technique	Frequency [Hz]			
	1st mode	2nd mode	3rd mode	4th mode
EFDD	2.225	4.543	7.867	16.590
SSI	2.218	4.540	7.872	16.610

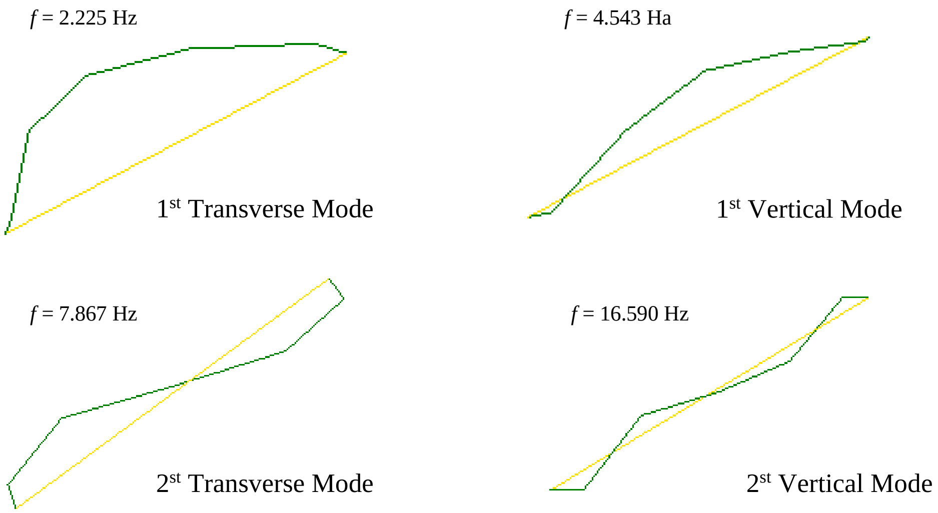

Fig. 8. The first four mode shapes of the PSC girder from OMA

5. Finite element modeling

To obtain the dynamic characteristics of the PSC girder, 3D FEM of the girder was created using the commercial software program SAP2000 [16]. The selected PSC girder is modeled with 275 frame elements and 24 tendons. The girder and strands were represented by frame and tendon elements, respectively. The 3D FEM of the girder created in this context is given in Fig. 9. To take into consideration, the changes of deformations and internal forces in the beam after the prestressing force is transferred to this type of girder, geometric nonlinearity should take into account in prestress loads case. Stiffness at the end of this case was used in modal analyses.

As a boundary condition, the left- and right-hand supports were selected as pinned and roller, respectively. Adjacent nodes between the frame elements and strands were connected to represent the perfect bond assumption. Transfer length was not taken into consideration. Self-weight of the girder was calculated from the finite element software directly. PF was calculated as 175 kN when the prestressing losses were taken into account and this force was simultaneously applied to all strands on both sides.

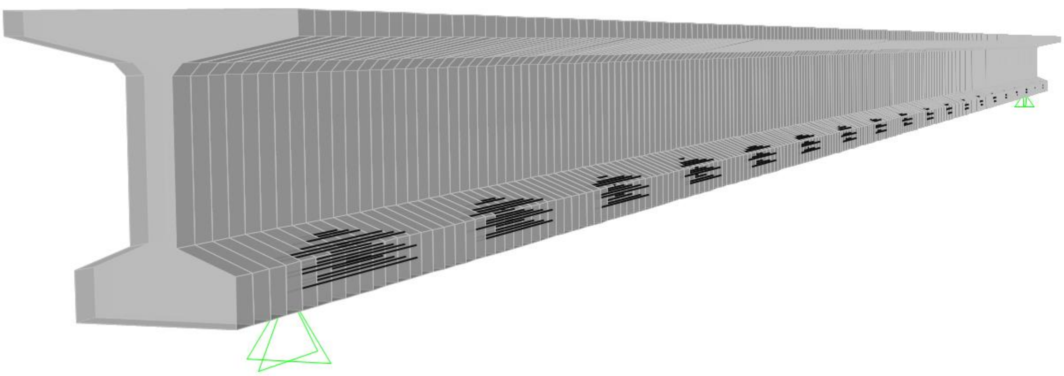

Fig. 9. Finite element model of the girder

6. Finite element analysis results

The first four MSs and NFs of the PSC girder obtained from the FEM modal analyses are shown in Fig 10. The numbers of four NFs of the girder were obtained which order between 1.89 and 15.29 Hz. The first four types of MSs of girder: transverse, vertical, transverse, and vertical modes, respectively.

The value of NFs obtained by an analytical formula suggested by Timoshenko et al. (1974), numerically by FEM and operational modal testing are given in Table 3.

It is seen that NFs obtained from analytical prediction and numerical by FEM where geometric nonlinearity was taken into account in the prestressing loads' case and stiffness at the end of this case was used in modal analyses are getting closer to each other. It is seen that the values obtained from the two methods mentioned are quite close to each other in all the given modes. The natural frequencies of PSC girder identify from EFDD and SSI techniques is not too far apart. When the results obtained from the analytical, numerical, and operational modal analysis used in determining the natural frequencies of the selected PSC girder are compared, it is seen that the values are quite close to each other. The differences between techniques used to identify the natural frequencies of PSC girder are fewer than 15%. From the results given in Table 3, it can be concluded that it is possible to obtain the modes and natural frequencies of the PSC girder by using only the formula suggested by Timoshenko without the laborious and long-term finite element method and operational modal analysis.

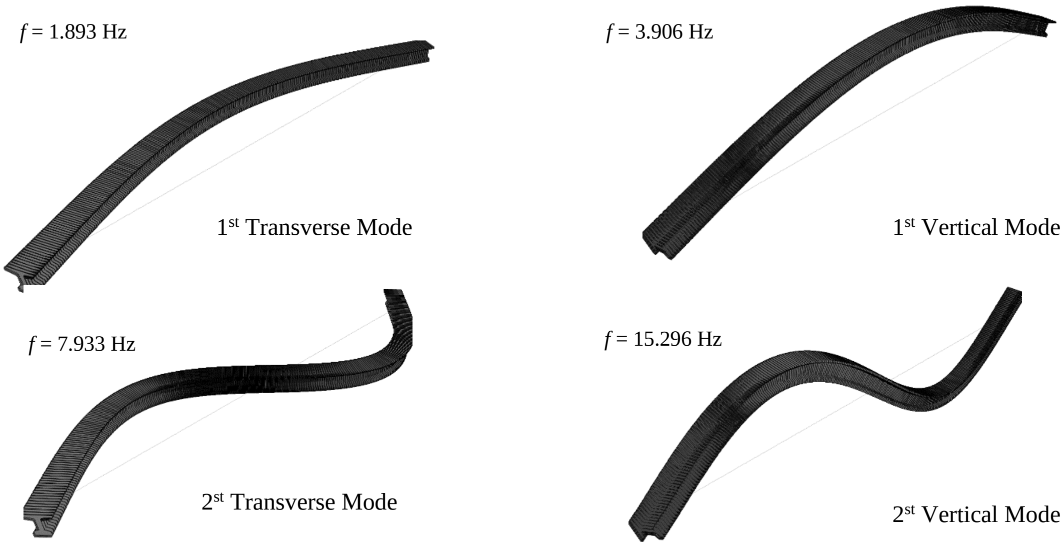

Fig. 10. MSs of the PSC girder obtained by the FEM.

Table 3. Natural frequencies of the first four modes of girder obtained using different techniques

Techniques	Frequency [Hz]			
	1st mode	2nd mode	3rd mode	4th mode
Analytical	1.937	3.958	8.667	16.303
Numerical	1.893	3.906	7.933	15.296
EFDD	2.225	4.543	7.867	16.590
SSI	2.218	4.540	7.872	16.610

7. Conclusions

The purpose of this paper is to obtain the modal parameters of prestressed concrete girder by analytical prediction suggested by Timoshenko et al. [10], numerically by FEM, and operational modal testing by OMA. For this purpose, one of a typical precast I-girder with 120 cm height and 27.45 m effective span length is selected as an application. The three-dimensional finite element model of the girder is modeled with SAP2000. The experimental measurements of the girder were conducted at the construction site by the operational modal testing method. For experimental measurements, ten uni-axial seismic accelerometers were mounted at the top flange of the PSC girder in the *x*- and *z*-direction. The vibrations that occur from the movement of trucks and cranes at the construction site and also the impact of the hammer were measured by these accelerometers as acceleration. The measured signal was collected at the data bank and then sent to the computer equipped with OMA software which used Enhanced Frequency Domain Decomposition and Stochastic Subspace Identification techniques. The dynamic behaviors of girders were derived from the analysis of this software. At the end of the study, mode shapes and natural frequencies obtained analytically, numerically, and experimentally were compared with each other. The main conclusions obtained from this study are:

- The mode shapes of the PSC girder obtained from FEM modal analysis and OMA are overlapping with each other.
- The analytical and numerical natural frequencies of the PSC girder are getting closer to each other.
- The natural frequencies and mode shapes of PSC girder identified by EFDD and SSI techniques are close to each other.
- The differences among techniques used to identify the natural frequencies of PSC girder are smaller than 15%.

This study concludes that dynamic characteristics of PSC girders such as natural frequencies and mode shapes obtained from theoretical prediction, FEM and operational modal testing are not too far apart.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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