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RESEARCH ARTICLE

Stability analysis of columns with variable cross-sections

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Abstract

In civil engineering applications columns with variable cross-sections are commonly used for various reasons and buckling has a very important role in the design of these members. In this investigation square columns with variable cross-sections and circular columns with variable cross-sections is considered. These examples evaluated for four different strength classes of normal strength concrete and four different boundary conditions. ANSYS Parametric Design Language codes are developed to analyze the columns with variable cross-sections models systematically and obtained results are presented with tables and graphics. It has been revealed that boundary conditions and the shape of the cross-section effects the stability of the columns dependent to the critical buckling load values.

Keywords

Stability analysis; Variable cross-section; Finite element method; ANSYS.

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1. Introduction

Columns are structural members that transmit vertical loads to the foundation in structural engineering. As seen in Fig.1, in civil engineering applications columns with variable cross-section are commonly used in order to reduce weight and increase strength, or to satisfy architectural needs. By using high strength materials structural members are becoming thinner and more slender. Against the loss of stability the structural members must be safe guarded also.

Buckling has a very important role in the design of these members. And also minimization of structural weight with maximization of critical buckling load is an important design problem for variable cross-section members.

The cross-sectional variation in structural members is used in order to provide the economy from the material and to lighten the structure in stability problems as in the stress problems. Variable cross-section structural members have less weight than uniform thickness ones.

Buckling loads are critical loads where structures become unstable. There are two kind solutions to perform a buckling analysis. One is eigenvalue buckling analysis, and the other is nonlinear buckling analysis. Eigenvalue buckling analysis computes the structural eigenvalues for the constraints and given system loading. This is also called classical Euler buckling analysis.

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Fig. 1. Column examples with variable cross-sections

In nonlinear buckling analysis to predict buckling loads; non-linear, large-deflection, static analysis is employed. Therefore this method is more accurate than eigenvalue analysis. [1].

Timoshenko and Gere are the father of modern engineering mechanics who wrote the best available guide to the elastic stability of structures. In their book the principles and theory of structural stability introduces[2] . Gursoy , solved buckling problems of elastic bar by using variational derivation and finite difference methods in his master thesis [3]. Karabalis and Beskos proposed a new numerical method for the stability analysis of linear elastic plane structures consisting of beams with constant width and variable depth [4]. Qiusheng et al. used Bessel functions and geometric series in their studies to analyze the buckling analysis of fixed and variable section bar systems under constant and variable axial load effects [5]. Gun, instigated an analysis of buckling of elastic straight bars using a new functional in his master thesis [6]. Pekbey et al., developed and designed an optimized composite column against buckling in their study and verified their solution with numerical analysis using ANSYS [7]. Li et al., made the stability analysis of a composite column under end force and distributed axial load in their studies. The composite column has variable material properties and varying cross-section and the integral equation method is formulated to model this problem [8]. Arbabi and Li, presented an integral-equation approach procedure for the axial buckling of variable cross-section elastic columns with step-varying profiles [9]. Eisenberger gives exact solutions for the buckling loads of variable cross-section members, loaded by variable axial force for several boundary conditions in his work [10]. Coşkun and Öztürk are investigated elastic stability analysis of Euler columns by using analytical approximate techniques [11]. Coşkun, used the Homotopy Perturbation Method for elastic stability analysis of tilt-buckled Euler columns with variable flexural stiffness [12]. Soltani and Sistani are applied the finite difference method (FDM) to investigate the buckling load of columns with variable flexural rigidity with different boundary conditions subjected to axial loads in their study [13]. Qiusheng et al. presented the exact solutions for stability analysis of bars with varying cross sections subjected to simple or complicated loads, including concentrated and variably distributed axial loads [14]. Sapalas et al. are studied a theoretical and a numerical analysis of tapered structural element subjected to an axial force and a bending moment in their paper [15]. Chen et al. are presented a new numerical method for evaluating the buckling loads of columns with varying cross-sections [16]. Eisenberger studied the exact solution of buckling loads for variable cross-section bars in a nonuniform thermal field [17]. Avcar investigated the elastic buckling of steel columns with three different cross sections and two different boundary conditions [18]. Darbandi et al. considered the static stability of the variable cross-section columns, subjected to distributed axial force in their study [19]. Yongjiu et al. investigate the buckling loads of a thin-walled box column with variable cross-section by using the approximate formulas based on the energy principle and the Galerkin's method [20]. Saracoglu and Uzun studied about buckling analysis of structural members with variable cross-sections and they investigate the effects of variable cross-sections by using finite element software ANSYS [21].

The purpose of this study is to investigate the effects of variable cross-section on the buckling load of the columns. In this study, a number of analyses are performed in ANSYS by using the developed APDL codes. This codes gives the calculation results practically for the systematic variation of the cross-section for the considered examples in the study. So that comparison for critical buckling loads between square varying columns and circular varying columns has been made easily.

2. Material and methods

The critical buckling load expression obtained for the prismatic column is stated below:

$$P_{cr} = \frac{\pi^2 E\, I}{\left(L_{ef}\right)^2} \tag{1}$$

In this equation E is modulus of elasticity, I is moment of inertia for the constant cross-section and L_{ef} is the effective buckling length.

Fig. 2 shows columns with four different boundary conditions as Clamped-Free (CF), Hinged-Hinged (HH), Clamped-Hinged (CH) and Clamped-Guided (CG) with effective buckling lengths. The cross-section of the columns are constant and the moment of inertia of the columns are also constant.

If the cross-section of the column is not constant, the critical buckling load will also be different depending on the variable cross-section [2]. In this case, the expression is as follows:

$$P_{cr}^{\text{var}} = \alpha \, \frac{\pi^2 E\, I_0}{\left(L_{ef}\right)^2} \tag{2}$$

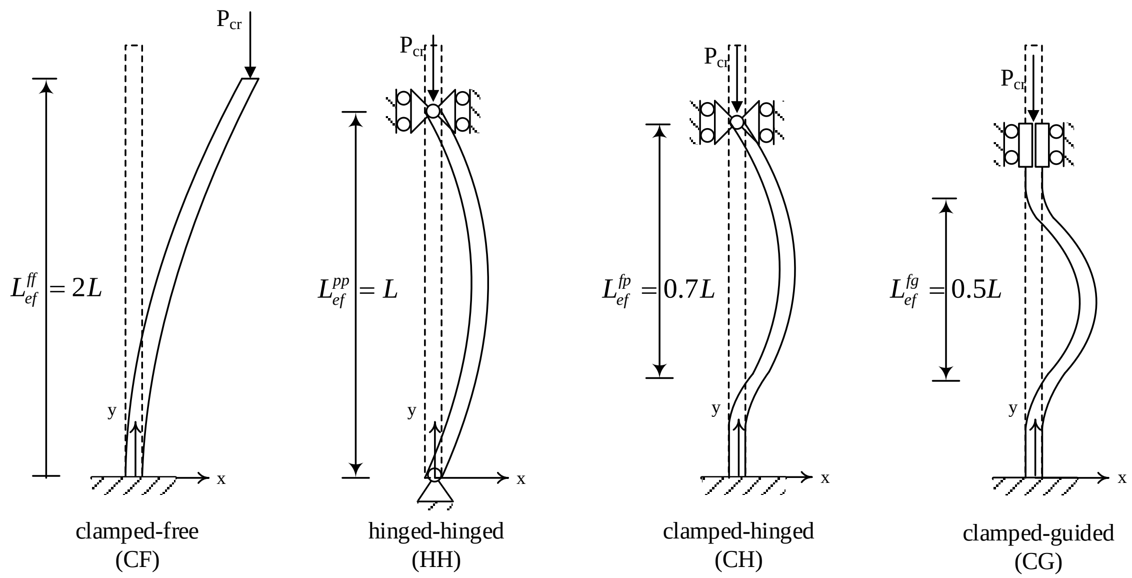

Fig. 2. Columns with different boundary conditions with effective buckling lengths.

In this equation, α is a numeric factor based on the ratio of the variable cross-sectional length to the total length of the column, E is modulus of elasticity, I_0 is moment of inertia for the constant part of the cross section and L_{ef} is the effective buckling length.

In this study, ANSYS 19.0 software were used to get the analytic results of buckling of columns with variable cross-sections. To perform an eigenvalue buckling analysis in ANSYS, prestress effects must be activated. To determine the critical buckling load in ANSYS, unit load must be used. Applying a load other than unit load will scale the answer by a factor of the load.

All of the buckling examples were analyzed for determining the critical buckling loads. The columns with variable cross-sections were modeled as a beam element (BEAM188) which is a linear (2-node) beam element in 3D with six degrees of freedom at each node. The degrees of freedom at each node include translations in *x*, *y* and *z*-directions and rotations about the *x*, *y* and *z*-directions [1].

With default settings, in BEAM188 element, six degrees of freedom occur at each node; these include translations in the *x*, *y* and *z*-directions and rotations about the *x*, *y* and *z*-directions (Fig. 3).

This element is suitable for analyzing elements with any cross-sections defined by using some special APDL commands. The beam elements are well-suited for linear, large rotation, and/or large strain nonlinear applications. The BEAM188 element is also suitable for analyzing slender to moderately stubby/thick beam structures. The element is based on Timoshenko beam theory which includes shear-deformation effects. And also warping of cross-sections is assumed to be unrestrained.

3. Numerical examples

In this study for investigating the effect of variable cross-section in buckling of columns two types of examples discussed. One of the problem has variable square cross-sections and the other one has variable circular cross-sections. All of the problems have the same constant length of L = 10 meters. To give a more general result for variable cross-section columns under buckling the same configuration in length for both square and circular columns were selected. The length of the L_1 in the varying part of the columns vary from 0 to 10 as shown in Fig. 4.

These examples evaluated for four different strength classes of normal strength concrete. Properties of these normal strength concrete is shown in Table 1.

Critical buckling loads were determined by using ANSYS and after the analyses numeric factors α are calculated from Eq. 2. While analyzing the problem, BEAM188 element in ANSYS library was selected.

In the finite element analysis, complex geometries are divided into simple elements with meshing process. Number of mesh elements affects the convergence, accuracy, and speed of the analysis. Significant amount of total time of the solution is depend on meshing of the problem.

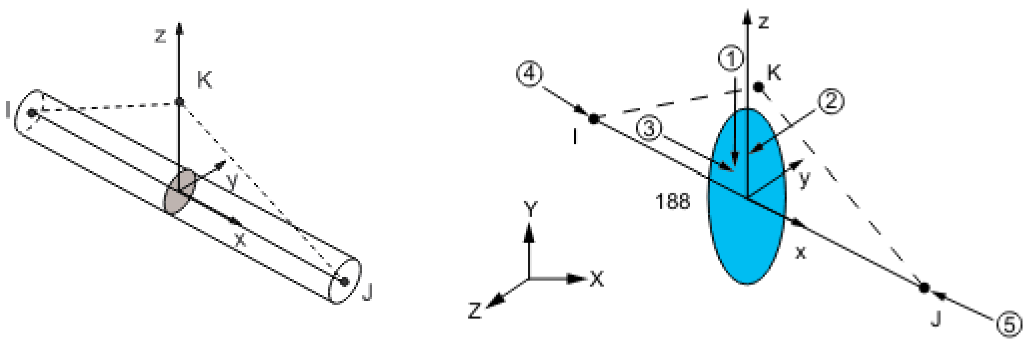

Fig. 3.. Geometry of BEAM188 finite element in ANSYS

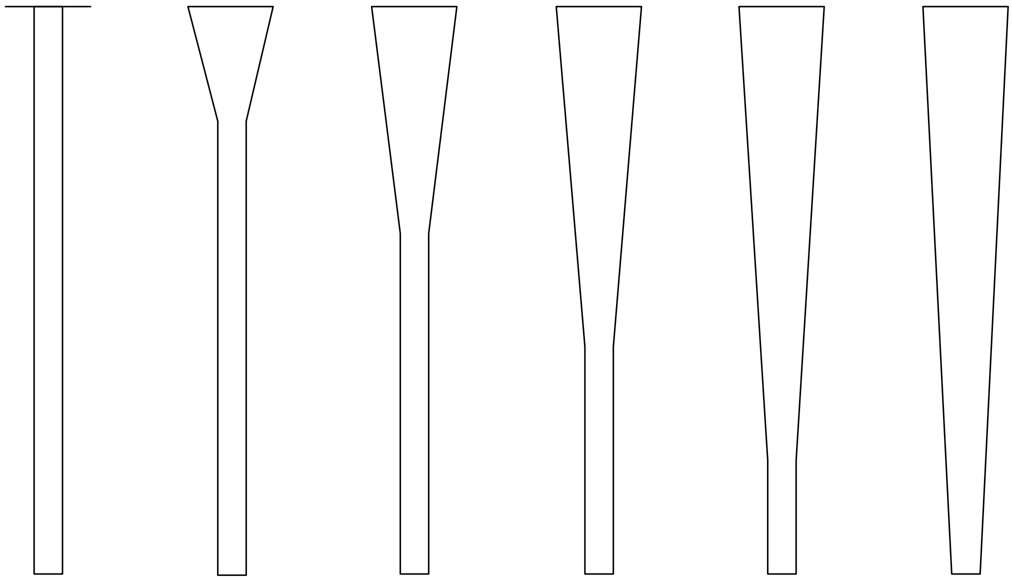

Fig. 4. Some examples of columns with variable cross-section according to L_1/L ratio

Table 1. Mechanical properties of normal strength concrete.

Strength class	C25/30	C30/37	C35/45	C40/50
Characteristic cylinder strength (f_{ck}, N/mm^2)	25	30	35	40
Characteristic cube strength (f_{ck}, N/mm^2)	30	37	45	50
Characteristic axial tensile strength (f_{ctk}, N/mm^2)	1.75	1.92	2.07	2.21
Modulus of elasticity (E, N/mm^2)	30250	31800	33227	34555
Shear modulus (G, N/mm^2)	12100	12720	13291	13822
Poisson's ratio	0.2	0.2	0.2	0.2
Coefficient of thermal expansion	1.00E-05	1.00E-05	1.00E-05	1.00E-05

This step is important for determining an accurate solution in finite element analyses. For this purpose a mesh refinement study is performed in this study. HH boundary condition is considered and graphs for critical buckling loads Pcr according to number of mesh elements for square and circular varying columns for L_1/L ratio is 0.50 is presented in Fig.5. From the graphs it can be seen that 100 mesh elements is the most suitable for the analyses. So that examples were modelled to the program by using 100 mesh elements.

3.1. Square columns with variable cross-sections

The total length of the column L and the length of the L_1 from the top varies to the smaller cross-section. As can be seen from Fig. 6, at the top of the column there is a square cross-section and one side of the square has a length a_1. Cross-section varies along to the length L_1 to the smaller and constant square cross-section and one side of the constant square cross-section has a length a_0.

In the square columns with variable cross-sections, length a_1 and a_0 is taken as 1,5m and 0,5m respectively as constants. The total length of the column was taken L = 10 meters as constant

Examples are modeled in ANSYS and critical buckling loads determined (Fig. 7). Constant α_s are calculated from Eq. (2).

Results for α_s coefficients and critical buckling loads in kN are presented for square varying columns in Table 2.

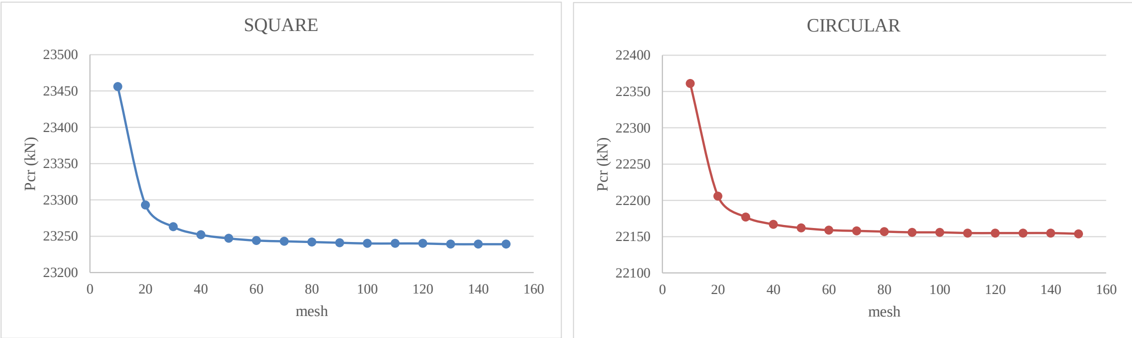

Fig. 5. Mesh refinement for varying columns with HH boundary condition.

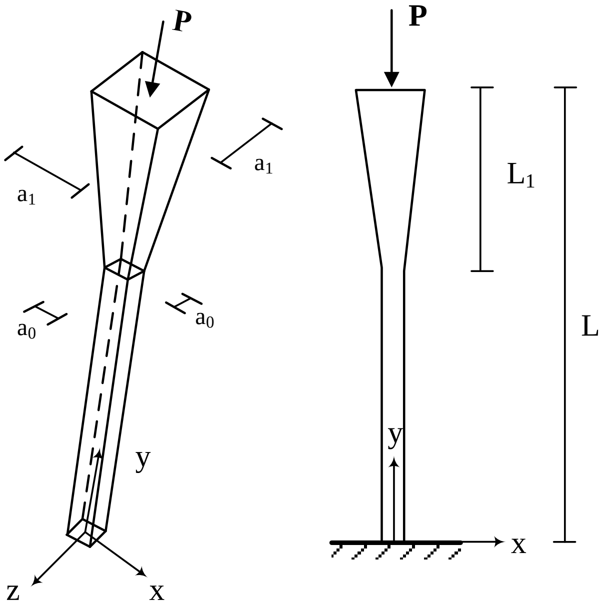

Fig. 6. Square column with variable cross-section

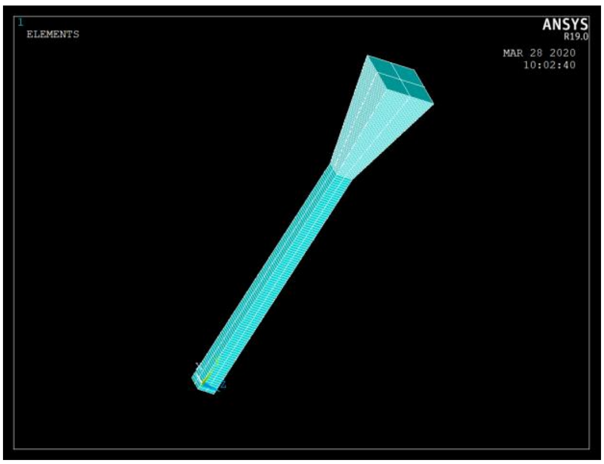

Fig. 7. ANSYS model for square columns

Table 2. Critical buckling loads P_{cr} (kN) and α_s coefficients for square varying columns

BC	L_1/L / Conc.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
CF	C25/30	3882	3887	3916	3993	4141	4390	4790	5430	6506	8523	13258
	C30/37	4081	4086	4117	4198	4353	4615	5035	5708	6840	8959	13938
	C35/45	4264	4269	4302	4386	4549	4822	5261	5964	7147	9361	14563
	C40/50	4434	4440	4474	4562	4730	5015	5471	6203	7432	9735	15145
	α	0.999	1.000	1.007	1.027	1.065	1.129	1.232	1.397	1.674	2.192	3.410
HH	C25/30	15461	15537	15963	16998	18906	22107	27422	36571	53375	85430	135720
	C30/37	16254	16333	16781	17869	19875	23240	28827	38446	56110	89808	142670
	C35/45	16983	17066	17534	18671	20767	24283	30121	40171	58628	93838	149070
	C40/50	17662	17748	18235	19418	21597	25254	31325	41776	60972	97590	155030
	α	0.994	0.999	1.027	1.093	1.216	1.422	1.763	2.352	3.433	5.494	8.728
CH	C25/30	31407	31722	33363	37073	43575	54158	71260	99161	142740	198130	271470
	C30/37	33016	33348	35073	38973	45808	56933	74911	104240	150054	208282	285380
	C35/45	34498	34843	36647	40721	47863	59487	78272	108918	156790	217626	298190
	C40/50	35877	36237	38111	42349	49777	61866	81402	113274	163055	226330	310110
	α	0.990	1.000	1.051	1.168	1.373	1.707	2.246	3.125	4.498	6.243	8.554
CG	C25/30	60810	72140	86840	106120	131150	160910	188350	217120	273710	372690	508830
	C30/37	63926	75837	91290	111560	137870	169150	198000	228240	287730	391790	534900
	C35/45	66795	79240	95390	116560	144050	176740	206880	238490	300640	409370	558910
	C40/50	69464	82410	99200	121220	149810	183800	215150	248010	312650	425720	581240
	α	0.978	1.160	1.396	1.706	2.109	2.587	3.028	3.491	4.401	5.992	8.181

3.2. Circular columns with variable cross-sections

In the second example cross-section of the column with variable cross-section is taken as circular (Fig. 8). Square columns with variable cross-sections is taken as a reference and circular columns with variable cross sections are equivalent to square ones. So that radius of the cross-sections are dependent with the size of the cross-sections of the square columns with variable cross-sections. In order to be the same volumes for all L_1/L ratio of the examples, radii of the circular cross-sections are taken as

$$r_i = \frac{a_i}{\sqrt{\pi}} \quad (i = 0,1) \tag{3}$$

Radius of constant cross-section part of structural member r_0 and the radius r_1 at the tips is taken dependent to a_0 and a_1 respectively. Total height of the column L is taken as 10m and the variable cross-section height L_1 is varies from 0 to 10 same as square columns with variable cross-sections (Fig. 4).

Examples are modeled in ANSYS and critical buckling loads determined (Fig. 9). Constant α_c are calculated from Eq. (2).

Results for αc coefficients and critical buckling loads in kN are presented for square varying columns in Table 3.

4. Results and discussion

From the results obtained it is possible to see that square varying columns and circular varying columns which has the same configurations and volumes, critical buckling loads and also α coefficient of the square varying columns are greater than the circular varying columns. Variation of α coefficients for varying columns are presented as a graph in Fig. 10. One can see from the Fig. 10 that when the L_1/L ratio is increasing the coefficients are increasing functionally for all boundary conditions. This means that when this ratio is increasing the volume of the column is also increasing so that critical buckling load is also increases.

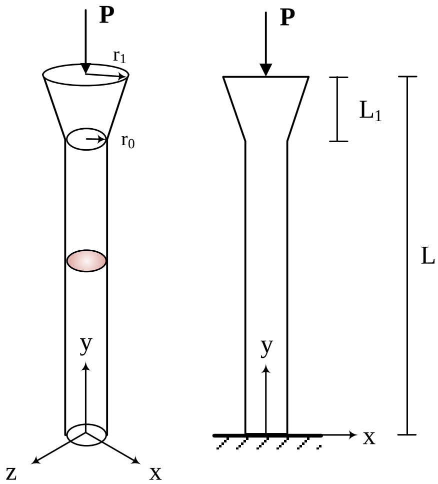

Fig. 8. Circular column with variable cross-section.

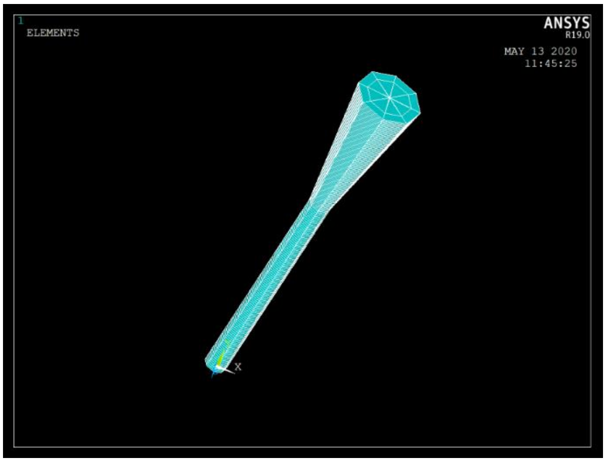

Fig. 9. ANSYS model for circular columns

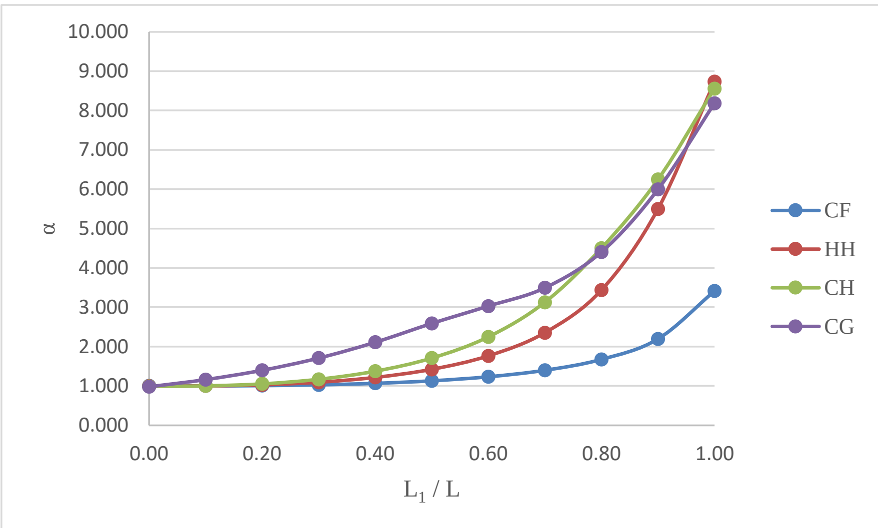

Fig. 10. Variation of α coefficients for varying columns.

Table 3. Critical buckling loads Pcr (kN) and α_c coefficients for circular varying columns.

BC	L_1/L / Conc.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
CF	C25/30	3700	3704	3732	3806	3946	4184	4564	5175	6200	8122	12635
	C30/37	3889	3894	3924	4001	4149	4398	4798	5440	6518	8538	13283
	C35/45	4064	4069	4100	4180	4335	4596	5014	5684	6811	8921	13879
	C40/50	4226	4232	4264	4347	4508	4779	5214	5911	7083	9278	14434
	α	0.997	0.998	1.005	1.025	1.063	1.127	1.230	1.394	1.670	2.188	3.404
HH	C25/30	14740	14811	15216	16204	18022	21076	26145	34875	50915	81537	129580
	C30/37	15495	15570	15996	17034	18946	22156	27485	36662	53524	85715	136220
	C35/45	16190	16269	16714	17799	19796	23151	28718	38308	55925	89563	142330
	C40/50	16837	16919	17382	18510	20587	24076	29866	39838	58161	93141	148020
	α	0.993	0.997	1.025	1.091	1.214	1.419	1.761	2.349	3.429	5.491	8.727
CH	C25/30	29954	30252	31817	35358	41565	51673	68018	94701	136434	189438	259570
	C30/37	31489	31802	33447	37169	43694	54320	71502	99552	143424	199143	272870
	C35/45	32902	33229	34948	38837	45655	56758	74711	104020	149860	208080	285120
	C40/50	34217	34557	36344	40389	47479	59025	77696	108176	155840	216393	296510
	α	0.988	0.998	1.050	1.167	1.372	1.705	2.245	3.125	4.502	6.251	8.566
CG	C25/30	58033	68865	82926	101382	125360	153880	180161	207702	262060	357206	487780
	C30/37	61007	72394	87175	106580	131784	161767	189390	218346	275486	375511	512770
	C35/45	63744	75642	91088	111361	137700	169027	197893	228144	287849	392363	535780
	C40/50	66292	78665	94727	115810	143200	175780	205800	237260	299350	408040	557200
	α	0.977	1.159	1.396	1.707	2.111	2.591	3.033	3.497	4.412	6.014	8.212

It can be seen from Tables 2 and 3 that when the cross-section ratio increases α coefficient is also increases.

For all of the boundary conditions, when the L_1/L ratio increases, α coefficient also increases functionally.

In cases where L_1/L is less than 0.70, the alpha coefficient is listed from small to large with CF, HH, CH and CG boundary conditions; however, if L_1/L is greater than 0.70, the alpha value obtained for the CG boundary condition is smaller than that of CH. Also, the difference between CF examples and others are getting greater. When the L_1/L ratio increases, the difference of α coefficient value between CF examples and other examples also increases.

5. Conclusion

Critical buckling loads for columns with variable cross-sections are different from prismatic columns with constant cross-sections because of their shape configurations. It is possible to see that columns with variable cross-sections are used in engineering structures. This type of geometry make the structural elements more stable under buckling. In such structures, as well as saving material a significant decrease in weight is also observed.

If the column ends are variable, the buckling length of the column decreases and the stability to buckling increases. From the examples, it can be seen that critical buckling load increases when the column section is variable. Basically two different examples considered in this study as square varying columns and circular varying columns. This problem investigated with four boundary conditions as CF, HH, CH and CG. Also, all of the critical buckling load calculations performed for four different concrete materials.

In this study, a number of analyses are performed in finite element software ANSYS with the use of developed APDL codes and critical buckling loads for the examples considered are determined. By using this buckling loads α coefficients are calculated from Eq. (2). The results are presented in tables and graphs. From the results it could be seen that in columns with variable cross-sections critical buckling loads varies dependent to the length ratio of the column. Length ratio is taken as varying length of the column to the total length of the column (L_1/L). As shown in the figures that presents the variation of α coefficients, although behavior of the problems are similar results are different.

As a result, boundary conditions and the shape of cross-section affect the stability of the columns dependent to the critical buckling load values.

This variable cross-section column problem can also be examined by taking different cross-sections. Furthermore, cross-sectional calculations can be made for the maximum critical load using optimization techniques.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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