

RESEARCH ARTICLE

Simplified irregular column analysis by equivalent square method

M.S. Al-Ansari, M.S. Afzal*

Qatar University, Department of Civil and Architectural Engineering, Doha, Qatar

Abstract

Equivalent square method is a simplified method to overcome the difficulty of analyzing irregular column sections, which has been a constant concern for a structural engineer, to design a safe and economical structure in modern buildings and bridge piers. Irregular column section with its equivalent square column section were both analyzed by finite element method based on ACI code of design. Eight irregular column sections were selected in this study to formulate their equivalent square sections. These sections were analyzed with the finite element software (SP-Column). The results obtained from the finite element method indicates that the equivalent square method is safe and reliable to use for the irregular column sections, selected in this study, with certain specified conditions.

Keywords

Irregular reinforced columns; Equivalent square method; SP-Column; Column analysis

Received: 24 February 2019; Accepted: 27 March 2019

ISSN: 2630-5763 (online) © 2019 Golden Light Publishing® All rights reserved.

1. Introduction

Columns are the vertical compression members, which transmit loads from the upper floors to the lower levels and to the soil through the foundations [1]. Based on the position of the load on the cross section, columns are classified as concentrically loaded, Figure 1, or eccentrically loaded, Figure 2.

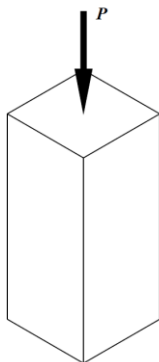


Fig. 1. Concentrically loaded columns

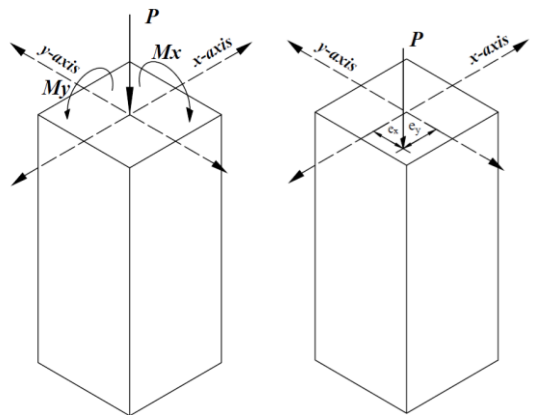


Fig. 2. Eccentrically loaded column

Eccentrically loaded columns are subjected to moments, in addition to axial force. The moments can be converted to a load P and eccentricity e_x and e_y . The moments can be uniaxial, as in the case when two adjacent panels are not similarly loaded,

* Corresponding author
E-mail: shekaib@qu.edu.qa

such as columns **A** and **B** in Figure 3. A column is considered biaxially loaded when the bending occurs about the x and y -axes, such as in the case of corner column **C** in Figure 3.

The strength of reinforced concrete columns is determined using the following principles:

1. A linear strain distribution exists across the thickness of the column
2. There is no slippage between the concrete and the steel
3. The concrete strain at failure for strength calculations is set equal to 0.003 mm/mm.
4. The tensile resistance of the concrete is negligible and disregarded.

The strength of reinforced concrete columns is usually expressed using interaction diagrams to relate the design axial load ϕP_n to the design bending moment ϕM_n [2]. Each point of the control points on the column interaction curve ($\phi P_n - \phi M_n$), represents one combination of design axial load ϕP_n and design bending moment

ϕM_n corresponding to a neutral-axis location, Figure 4, [3]. The interaction diagram is separated into a tension control region and a compression control region. The balanced condition occurs when the failure develops simultaneously in tension (i.e., steel yielding) and in compression (concrete crushing).

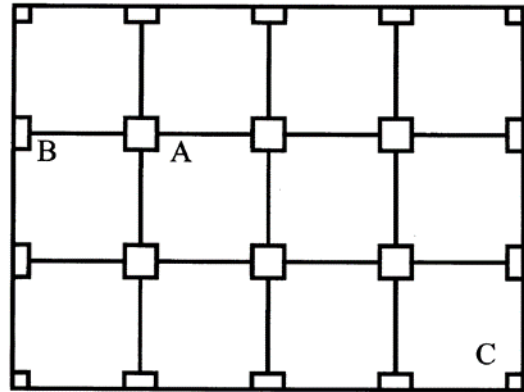


Fig. 3. Uniaxially and biaxially loaded column

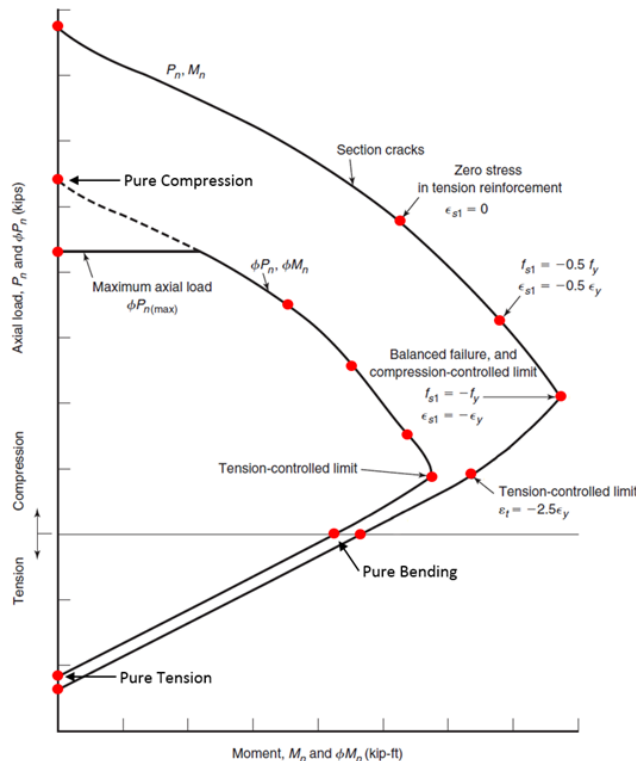


Fig. 4. Control points for column interaction curve ($\phi P_n - \phi M_n$) [3]

The manual design of reinforced concrete columns is usually performed using hand-computation procedure that checks whether the point (P_u, M_u) , which is defined by the factored axial load P_u and the factored bending moment M_u , is inside, outside, or on the interaction diagram $(\phi P_n - \phi M_n)$. The strength of the column is not adequate if the point (P_u, M_u) is outside the curve $\phi P_n - \phi M_n$. The closer is the point to the curve, the more economical is the design. Further details on the reinforced column analysis and design can be found elsewhere [4-8].

2. Equivalent square method

The present study is conducted to check the adequacy of the equivalent square sections for different irregular shaped sections in accordance with the ACI code of design [9] using the finite element software (SP-Column) [10]. This will help to analyze these irregular column sections with quite an ease by analyzing their equivalent square sections. Previous research studies for the irregular shaped sections were only limited to certain irregular shapes [11-15]. Eight different irregular shaped column sections were selected in this study as they are quite often used in the construction Industry. The dimensions of the equivalent square column are first determined by equating the areas of the given irregular column with that of its equivalent square column. The reinforcement ratio (ρ) should be the same in both sections. The equivalent square section of selected Irregular columns is presented in Table 1.

Using the above dimensions for the irregular sections, equivalent formulas for square sections were formulated and based on these formulas any irregular section (C1-C9) with different dimensions can be analyzed.

3. Numerical examples- Equivalent square section

In this study, the irregular column shape sections as well as the equivalent square sections were analyzed by using the finite element software (SP Column). These irregular sections were given some initial dimensions and their equivalent square section were obtained using these above equivalent formulas. Both sections are having the same area of steel which means having the same reinforcement ratio (ρ) (Table 2). The input data for analysis of the above eight columns (C-1 to C-8) are listed in Table 3 where f_c' and f_y are the concrete compressive strength and steel yield strength respectively. Moreover, P_u is the ultimate axial load applied on the column and M_{ux} and M_{uy} are the ultimate applied moments in x and y directions. The equivalent square section of column C-9 will have similar dimension (h) as of irregular section and this will give similar results in terms of column capacity. Therefore, the analysis of this column is not included in this present study.

The above eight column sections along with their equivalent square sections (C1 – C8) were analyzed using the SP- Column software and the results obtained are illustrated in Table 4. P_c is the axial load capacity of the column and M_{cx} and M_{cy} are the moment capacities in x and y direction respectively.

The columns C-4, C-6 and C-7 were analyzed as uniaxial column while the remaining columns were solved as biaxial column with the moments in X and Y directions respectively. So, the equivalent square section of hexagonal, triangular and trapezoidal sections showed promising results with a difference of 6%, 10% and 2% respectively. This reflects that these equivalent square sections (C-4, C-6 and C-7) are safe to use. For the triangular shaped irregular section (C-6) with the equal as well as for unequal legs dimensions, the equivalent square section for both of them gave good results. Moreover, these sections can also work as biaxial columns but in this study, they are limited with moments acting only in one direction.

Table 1. Equivalent square section

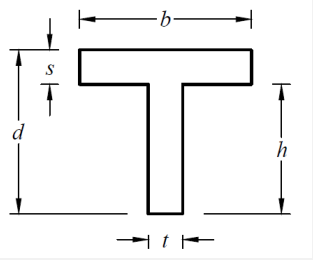
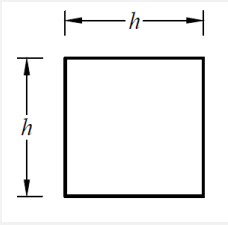
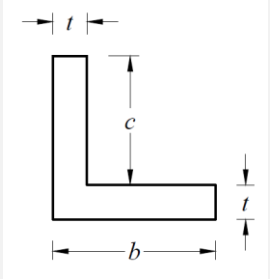
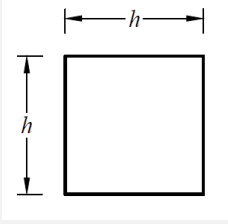
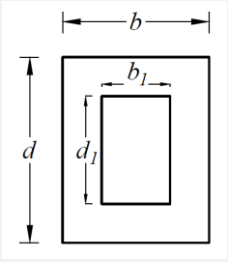
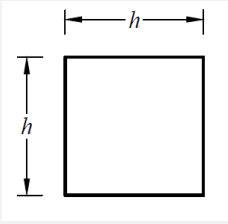
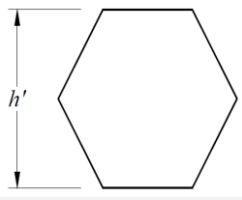
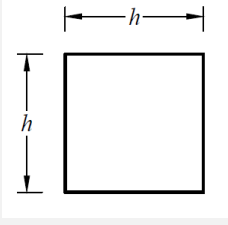
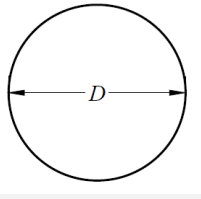
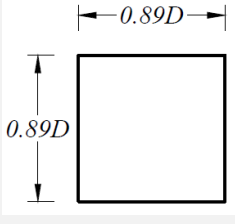
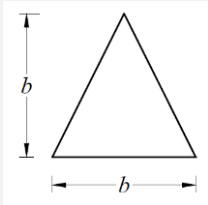
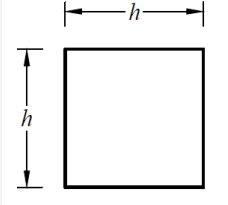
Column	Irregular Column Shapes	Equivalent Square Section	Equivalent Formula
C-1			$h = \sqrt{b(s) + h(t)}$
C-2			$h = \sqrt{t(b + c)}$
C-3			$h = \sqrt{bd - b_1d_1}$
C-4			$h = 0.93h'$
C-5			$bh = \frac{\pi}{4}(D^2) \cong b = h$ $h = \sqrt{\frac{\pi}{4}}D$
C-6			$h = \frac{b}{\sqrt{2}}$

Table 1. Continued

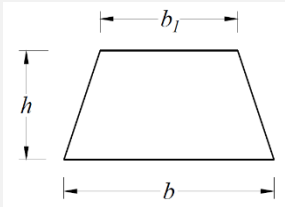
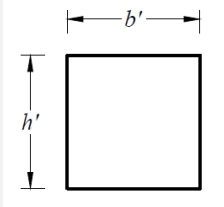
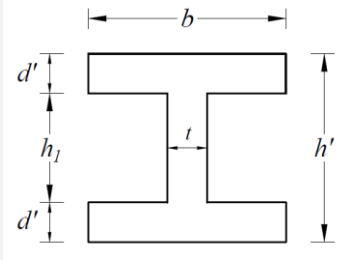
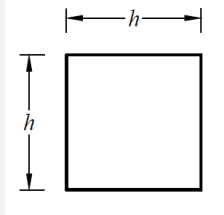
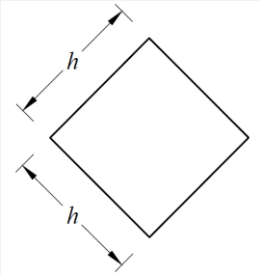
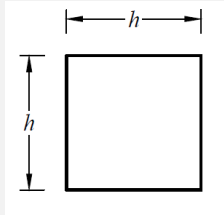
C-7			$b' = \frac{b + b_1}{2}, h = h'$ or for square $h' = \sqrt{\left(\frac{b + b_1}{2}\right)h}$
C-8			$h = \sqrt{bh' - h_1(b - t)}$
C-9			$h = h$

Table 2. Equivalent square section examples

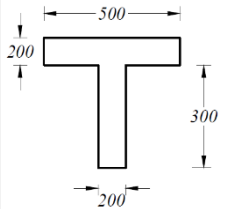
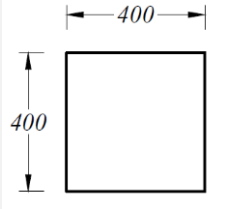
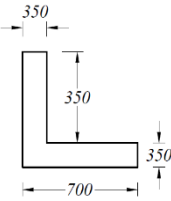
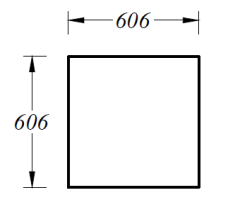
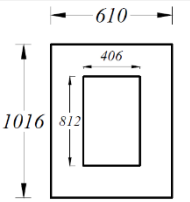
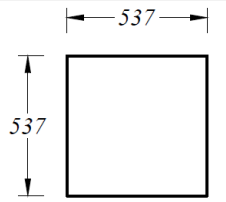
Column No.	Irregular Section		Equivalent Section	
	Shape	As (mm ²)	Square	As (mm ²)
C-1		8φ25		8φ25
C-2		8φ30		8φ30
C-3		28φ16		28φ16

Table 2. Continued

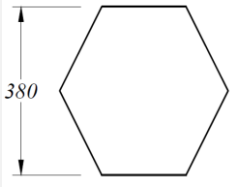
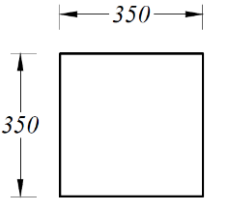
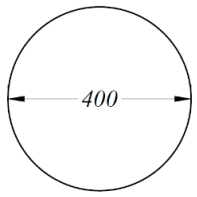
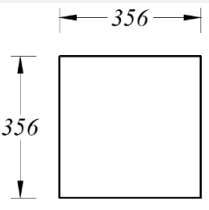
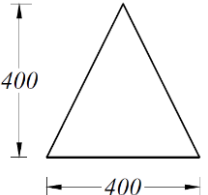
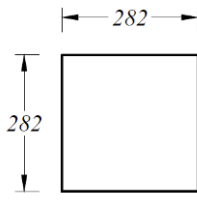
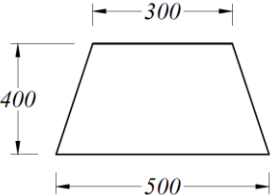
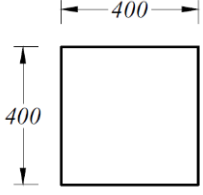
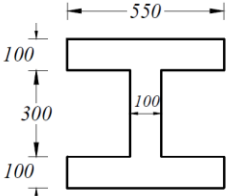
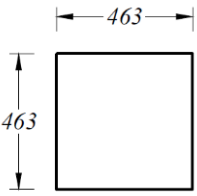
C-4		6 ϕ 20		6 ϕ 20
C-5		12 ϕ 32		12 ϕ 32
C-6		6 ϕ 25		6 ϕ 25
C-7		8 ϕ 20		8 ϕ 20
C-8		6 ϕ 25		6 ϕ 25

Table 3. Input data for column design

Column No.	f'_c (MPa)	f_y (MPa)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	ϕ_c
C-1	30	400	900	150	75	0.7
C-2	30	400	1800	500	200	0.7
C-3	30	415	3000	300	300	0.65
C-4	30	400	500	100	-	0.65
C-5	30	415	1120	90	180	0.65
C-6	30	400	1000	70	-	0.65
C-7	30	415	2500	110	-	0.65
C-8	30	420	1400	350	100	0.65

Table 4. Column analysis results

Column	Irregular Section			Equivalent Square Section		
	P_c	M_{cx}	M_{cy}	P_c	M_{cx}	M_{cy}
C-1	1580	263	132	1260	209	104
C-2	2136.6	594	237.5	2064.6	573	229
C-3	5316	531	531	3027	302	302
C-4	643	128.6	-	688	137	-
C-5	1308	105	210	1154	93	185
C-6	1302	90	-	1165	81	-
C-7	2780	122.3	-	2750	121	-
C-8	1703	243	121	1780	254	127

The remaining irregular sections were analyzed as biaxial column sections and their equivalent square section results vary from section to section. The results for the T and L shape sections (C1 and C-2) showed that the equivalent square section results are quite close to the original irregular column section. However, the results of these T and L shaped irregular sections are only reliable as long as both the legs are having equal dimensions. For the unequal dimensions of these sections, the equivalent square section results are unpredictable and are not safe to use.

The results for the tube section (C-3) showed that tube section is much stronger than the equivalent square section in terms of load as well as in moment capacity in both directions. This shows that the equivalent square section of the tube-shaped column is very safe and conservative for both axes.

The equivalent square section for circular section of any diameter can be found by the relationship $h=0.89D$ and this equivalent square section always provide conservative and safe results. The results for the I shaped section (C-8) shows that it is much stronger with its equivalent square section in the major axis but the section is weak with respect to its minor axis. The results for the axial load capacity as well as the moment capacity for the biaxial columns are illustrated in the bar charts (Figures 5 and 6) respectively.

The sample result output of column C-8 (I-shaped section) from the (SP-column) is also displayed for the reference in Figure 7.

4. Numerical examples for hollow sections

Some irregular sections with a circular opening in the middle were also analyzed in this research study. Four column sections (C-4-H, C-5-H, C-6-H and C-7-H) were selected to be analyzed as hollow sections. Two of the column sections (C-4-H and C-5-H) were analyzed as biaxial columns while the remaining two sections (C-6-H and C-7-H) were solved as uniaxial sections. The dimensions provided to these hollow sections are shown in Table 5. The input data for these selected hollow sections is provided in Table 6.

These hollow column sections were analyzed using the finite element software (SP-Column) and the results obtained for the irregular and equivalent square section are depicted in Table 7. The sample result output of column C-4-H (Hexagonal shaped section with circular opening) from the (SP-column) is also displayed for the reference in Figure 8.

The equivalent solid square section for the all the irregular hollow shape sections provide the safe and conservative results. The comparison between the results obtained from the irregular section to the ones obtained through equivalent square sections are also represented in bar charts (Figure 9 for the axial load capacity results and Figure 10 for the moment capacity respectively).

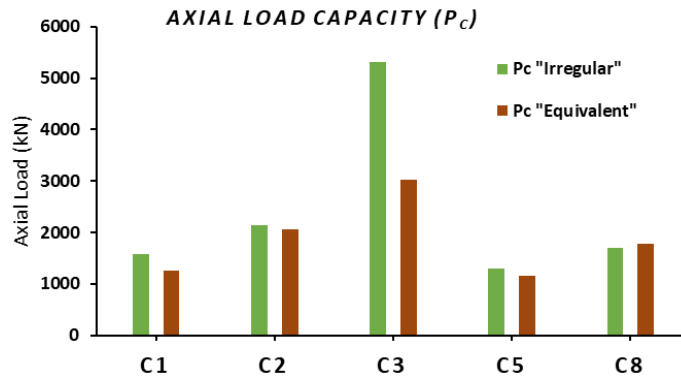


Fig. 5. Axial Load capacity comparison (biaxial columns)

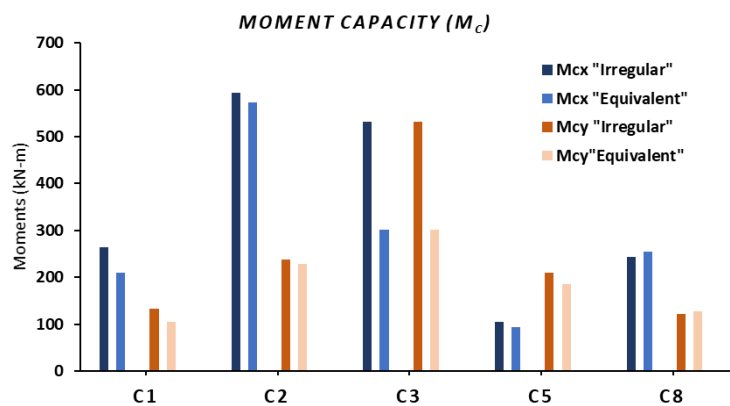


Fig. 6. Moment capacity comparison (biaxial columns)

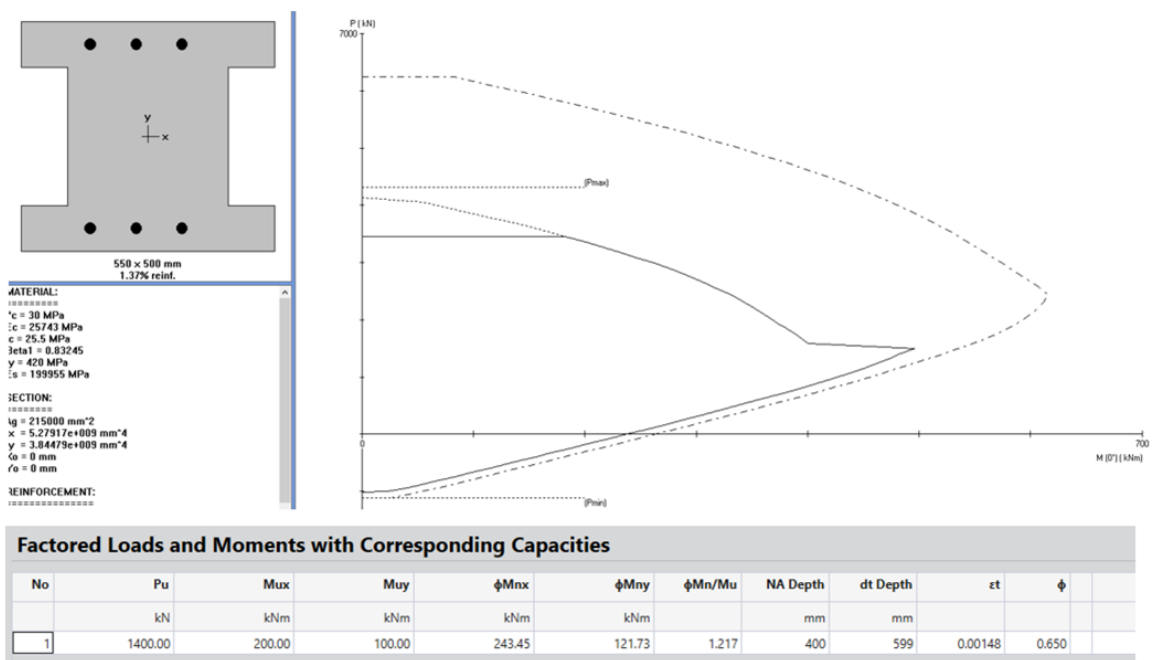


Fig. 7. Result output of SP-Column (C-8, I-shaped column section)

Table 5. Sections with opening (examples)

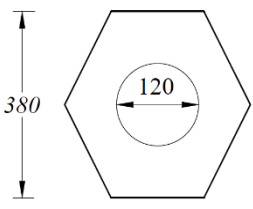
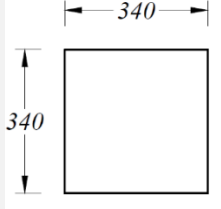
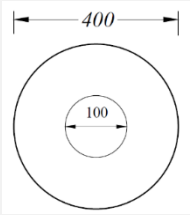
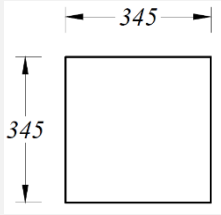
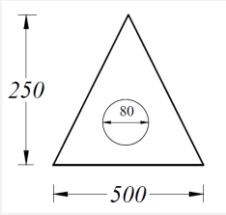
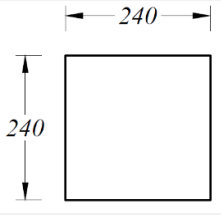
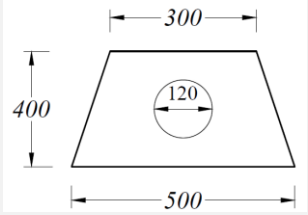
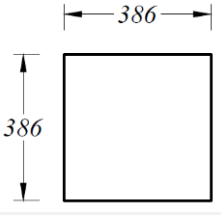
Column No.	Irregular Section with Opening		Equivalent Section	
	Shape	A_s (mm ²)	Square	A_s (mm ²)
C-4-H		6 ϕ 20		6 ϕ 20
C-5-H		12 ϕ 32		12 ϕ 32
C-6-H		6 ϕ 25		6 ϕ 25
C-7-H		8 ϕ 20		8 ϕ 20

Table 6. Input data for hollow sections

Column No.	f'_c (MPa)	f_y (MPa)	P_u (kN)	M_{ux} (kN.m)	M_{uy} (kN.m)	ϕ_c
C-4-H	30	400	500	70	60	0.65
C-5-H	30	415	1120	90	160	0.7
C-6-H	30	400	200	50	-	0.65
C-7-H	30	415	2300	1100	-	0.65

Table 7. Hollow section column analysis results

Column	Irregular Section			Equivalent Square Section		
	P_c	M_{cx}	M_{cy}	P_c	M_{cx}	M_{cy}
C-4-H	594	83.13	71.25	510	71.43	61.23
C-5-H	1422	114.3	203.2	1149	92.4	164.2
C-6-H	326	81.65	-	243	61	-
C-7-H	2595	124.1	-	2443	116.8	-

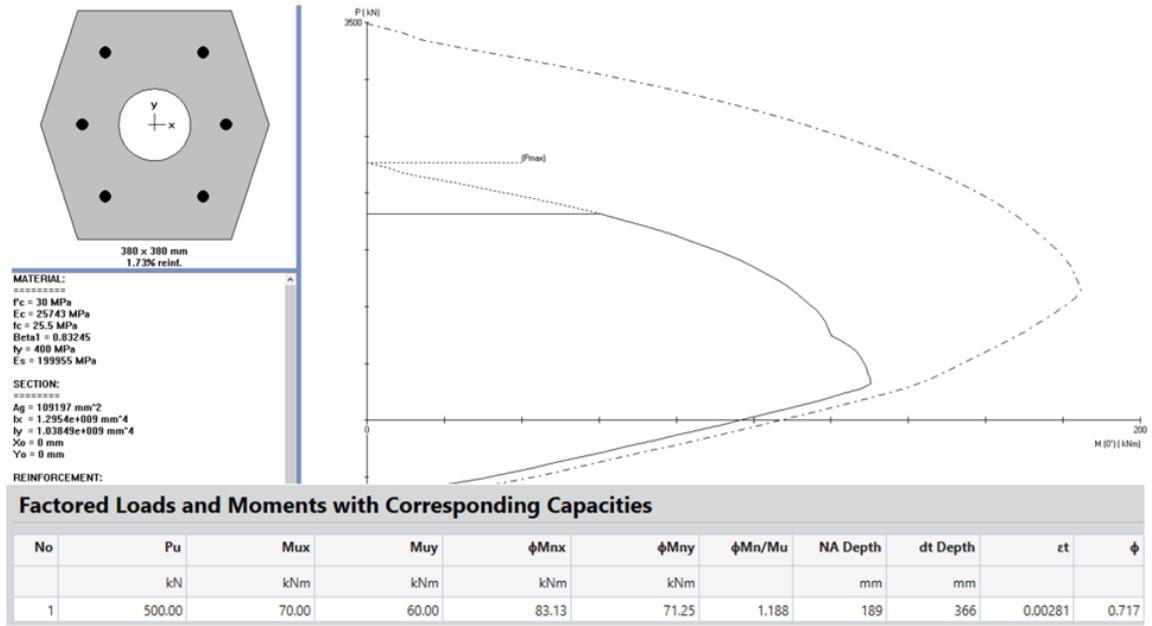


Fig. 8. Result output of SP-Column (C-4-H, Hexagonal column section with circular opening)

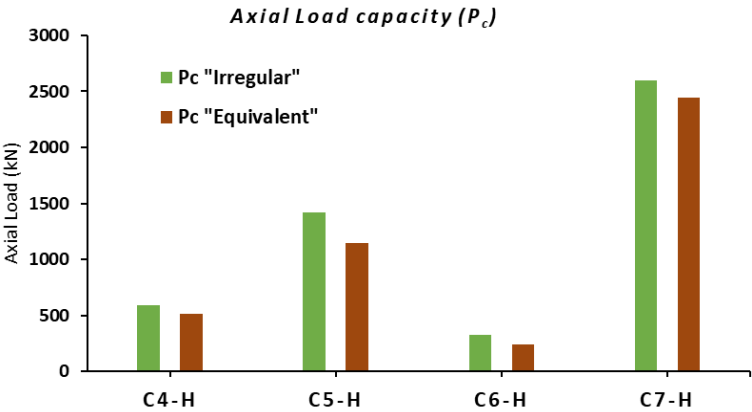


Fig. 9. Axial Load capacity comparison (hollow sections)

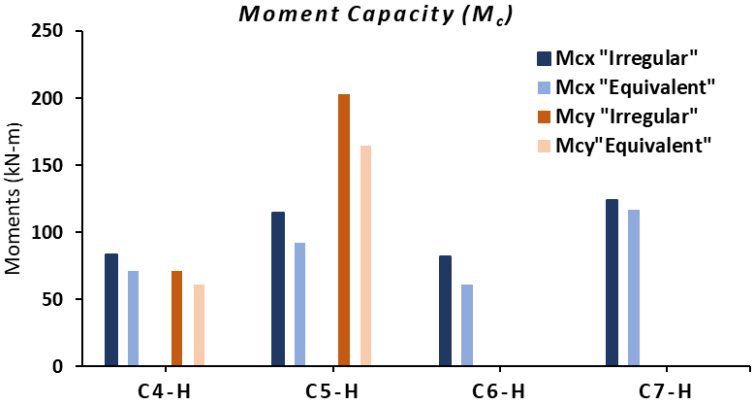


Fig. 10. Moment capacity comparison (hollow sections)

5. Conclusion

Equivalent square method is a simplified method to overcome the difficulty of analyzing irregular column sections. The agreement between irregular section and the square section finite element analysis results indicate that the equivalent square method is reliable, and it concludes to following points.

- 1- In general, the equivalent square section for all the irregular sections (*solid as well as hollow sections*) are conservative and safe to use.
- 2- The I-shaped section is stronger than the equivalent square section with respect to the main axis, but it is weaker with respect to the minor axis.
- 3- The tube section is stronger than equivalent square section with respect to both axes.
- 4- L and T shaped section works only if they have equal leg dimensions. For unequal leg dimensions, the results are not safe.
- 5- Triangular shaped sections (solid and hollow) with equal or unequal legs gives satisfactory results.
- 6- Equivalent square section of irregular sections with equal sides and symmetrical reinforcement gave good results and it is reliable. Equivalent square section of Irregular sections with unequal sides and unsymmetrical reinforcement must not be used with variation more the 10%.

References

- [1] Al-Ansari MS, Ahmed BS (1999) MATHCAD: Teaching and Learning Tool for Reinforced Concrete Design. International journal of Engineering Education 15(1): 64-71.
- [2] Rodriguez JA, Aristizabal-Ochoa JD (1999) Biaxial interaction diagrams for short RC columns of any cross section. Journal of Structural Engineering ASCE 125(6).
- [3] <https://www.structurepoint.org/pdfs/Interaction-Diagram-Tied-Reinforced-Concrete-Column-Symmetrical-ACI318-14.htm>, (Accessed: 11 November 2018).
- [4] Rafiq MY, Southcombe C (1998) Genetic algorithms in optimal design and detailing of reinforced concrete biaxial columns supported by a declarative approach for capacity checking. Journal of Computers and Structures 69(4).
- [5] McCormac JC, Russell HB. Design of reinforced concrete. Wiley, 10th Edition, USA, 2015.
- [6] Bresler B (1960) Design criteria for reinforced columns under axial load and biaxial bending. ACI Journal Proceedings 57(11).
- [7] Guo Z. Principles of reinforced concrete. Butterworth-Heinemann, 2014.
- [8] Subramanian N. Design of reinforced concrete structures. Oxford University Press, 2014.
- [9] ACI-318-14. Building code requirements for structural concrete. American Concrete Institute, ISBN: 978-0-87031-930-3, USA, 2014.
- [10] SP Column v6.00. Design and investigation of reinforced concrete column sections. 5420 Old Orchard Rd Skokie, IL 60077, USA, 2016.
- [11] Ramamurthy LN, Hafeez Khan TA (1983) L-shaped column design for biaxial eccentricity. Journal of Structural Engineering, ASCE 109(8).
- [12] Wsu CTT (1985) Biaxially loaded L-shaped reinforced concrete columns. Journal of Structural Engineering, ASCE 111(12).
- [13] Mallikarjuna, Mahadevappa P (1992) Computer aided analysis of reinforced concrete columns subjected to axial compression and bending-I L-shaped sections. Computers & Structures 44(5): 1121-1138.
- [14] Sfakianakis MG (2002) Biaxial bending with axial force of reinforced, composite and repaired concrete sections of arbitrary shape by fiber model and computer graphics. Advances in Engineering Software 33(4): 227-242.
- [15] Dahiya N, Sehgal KV, Saini B (2016) Analysis and design of rectangular and L-shaped columns subjected to axial load and biaxial bending. International Journal of Structural Engineering and Analysis 2: 15-22.