RESEARCH ARTICLE

Effect of fuzzy-membership function types on the search capability of interactive search algorithm

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Abstract

Fuzzy logic is widely used as the fundamental platform for the many artificial intelligence-based systems like control of subways, air conditioners, transmission systems, facial pattern recognition, antiskid braking systems. In the theory of fuzzy set, membership functions (MFs) provide an infrastructure for determining the degree of truth in a fuzzy model. So, they can play important role in the performance of these logical mechanisms. Different types of MF can be defined to fuzzification-defuzzification process (i.e. converting Boolean input to a fuzzy output and vise-versa). They are categorized and named based on the shape of their diagrams as triangular, Gaussian, trapezoidal and so forth. In the current study the effect of MF's type on the search capability of Interactive Search Algorithm (ISA) is assessed. To meet this aim, a metaheuristic technique that are known as Interactive Search Algorithm and strengthen with fuzzy adjustment mechanism is tested on solving a number of benchmark problems involving different types of MFs. The results indicate that due to stochastic essence of metaheuristic approaches the type of MF does not seriously affect the search capability of this technique. Therefore, it can be stated that the MFs should be selected based on the simplicity criterion. In other words, the simplest MF which provides the designers requirements can be the best choice.

1. Introduction

Most of the engineering optimization problems have a complex search space and generally finding a continuous and differentiable objective function for this class of problems is very difficult or mostly unattainable [1]. So, applying the gradient-based optimization methods which require the continuous (or at least partially continuous) objective function(s) and its different order gradients mostly is very challenging or even impossible in some cases [2-5]. Consequently, alternative optimization tool is required for such a complex problem.

Metaheuristic algorithms are the non-gradient based algorithms which do not require any continuous objective function and/or its gradients. These methods are generally put forward a simple stochastic mathematic algorithm which is inspired from the natural phenomenon like physical and natural rules, social behaviors or collaboration in the fish and/or birds colonies [6-23]. In addition of these specifications their ease to implementation makes them applicable in solving a wide range of optimization problems [24-28].

One can list some of these methods as, differential-based harmony search algorithm for the

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optimization of continuous problems, Search Group Algorithm (SGA) [23], Virus optimization algorithm (VOA) [14], Multi-strategy adaptive particle swarm optimization [18]. Each of these methods based on the governing conditions of the selected optimization problems has its own strengths and weaknesses. One of the most important factors which affects the search capability of the metaheuristic technique is its capability to adequately and properly perform exploitation and exploration search behaviors. In this regard, an algorithm that can adapt itself more closely to the dominant conditions of the problem shows higher search capability.

The metaheuristics mostly uses internal adjustable parameters to perform this action. The appropriate values for these parameters can be set via test and training sets or by applying auxiliary tuner modulus. In the current investigation the fuzzy mechanism is used to adjust the internal parameters of the Interactive Search Algorithm (ISA) introduced by Mortazavi et al. [29]. This algorithm uses two main internal adjustable parameters to balance the search behavior of the algorithm. Similar to several other optimization algorithms these internal terms highly affect the search capability of the technique. To maximize the search performance of the algorithm these parameters should be adjusted for desired class of problem(s) through the proper sensitivity analyses [30]. However setting up the internal parameters for the specific type(s) of problem may cause the technique to decrease its performance on solving other type of problems [31]. To prevent this shortcoming, the smart decision-making systems can be integrated to these methods. In this regard the fuzzy logic is applied as an auxiliary system which permanently monitors the optimization process and adaptively sets the proper values for internal coefficients of the proposed ISA method. To form such system different membership functions (MFs) can be implemented. The current study investigates the effect of different type of membership functions on the search performance of the ISA method. In this regard, six well known

membership functions named as Triangular, Trapezoid, two variants of Gaussian, Gbellmf, and Sigmoidal are selected to test the effect of the membership functions on the search performance of the ISA algorithm. There several mathematical functions and mechanical problems are chosen as benchmark problems. At the following firstly the applied ISA method is briefly described and then the membership functions are explained. In the next section the numeric tests are perfumed and results are reported. Finally, in the conclusion part the attained outcomes are instructed.

2. Interactive search algorithm (ISA)

Interactive Search Algorithm (ISA) is the hybrid optimization algorithm introduced by Mortazavi et al. [29]. This method utilizes two different patterns to navigate agent in the search space. They are named as *tracking* and *interacting* patterns. In the tracking pattern ith each agent (X_i) is updated considering three specific agent as weighted agent (X^W), the global best agent (X^G) and the prior best position stored in the memory of randomly selected another agent (X_j^P). In the interacting pattern, the current particle updates its position via pairwise data sharing with other agents in the colony. In each iteration agents based on their tendency factor select either interacting or tracking paradigm. The tendency factor (τ_i) in the standard ISA is designated to each agent randomly from [0,1] interval with normal distribution. It should be noted that the tracking and interacting patterns provide the global and local search requirements of the ISA algorithm, respectively. based on given definitions the proposed ISA algorithm mathematically is defined as below:

if $\tau_i \geq 0.3$

$$\left.\begin{aligned}
{}^{t+1}\mathbf{v}_i = w_i \times {}^t\mathbf{v}_i &+ diag\left(\boldsymbol{\varphi}_{1i} \otimes \left({}^t\mathbf{X}_j^P - {}^t\mathbf{X}_i\right)\right) \\
&+ diag\left(\boldsymbol{\varphi}_{2i} \otimes \left({}^t\mathbf{X}^G - {}^t\mathbf{X}_j^P\right)\right) \\
&+ diag\left(\boldsymbol{\varphi}_{3i} \otimes \left({}^t\mathbf{X}^w - {}^t\mathbf{X}_j^P\right)\right)
\end{aligned}\right\} \;(Tracking)$$

$$(1a)$$

if $\tau_i < 0.3$

$$\left. \begin{array}{l} {}^{t+1}\mathbf{v}_i = diag\left(\mathbf{r}_i^T \otimes \left(\mathbf{X}_i - \mathbf{X}_j\right)\right) \text{ if } f(\mathbf{X}_i) < f(\mathbf{X}_j) \\ {}^{t+1}\mathbf{v}_i = diag\left(\mathbf{r}_i^T \otimes \left(\mathbf{X}_j - \mathbf{X}_i\right)\right) \text{ if } f(\mathbf{X}_i) > f(\mathbf{X}_j) \end{array} \right\} \left(Interacting\right) \qquad (1b)$$

$${}^{t+1}\mathbf{X}_i = {}^{t}\mathbf{X}_i + {}^{t+1}\mathbf{v}_i \qquad (1c)$$

where X_i is the ith agent (i.e. vector) in the colony, $X_i = \left[x_i^1, x_i^2, x_i^3, ..., x_i^{nd}\right]$ and "t" and "$t+1$" superscripts are the current and updated conditions, respectively. Also, $\otimes$ indicates the Kronecker multiplier and $diag(\ )$ returns a vector holds any square matrix's diagonal elements. To provide more clarity on the mechanism of this multiplication assume two n-dimensional vectors namely $\mathbf{a} = \left[a_1, a_2, ..., a_n\right]$ and $\mathbf{b} = \left[b_1, b_2, ..., b_n\right]$, for these vectors $diag\left(\mathbf{a} \otimes \mathbf{b}\right)$ returns an n-dimensional vector which their elements are $a_l b_l$ where $1 \le l \le n$. Also, $1 \le i, j \le np$, $i \ne j$ and np indicates the size of population. For ith agent $\varphi_{ki} = C_k \times rand_{ki}$ for $k \in \{1,2,3\}$ and $C_1 = -(\varphi_{2i} + \varphi_{3i})$, $C_2 = 2$, $C_3 = 1$ are coefficients of acceleration, and $rand_{ki}$ are vectors hold random numbers selected from the [0, 1] interval. ${}^{t}\mathbf{X}^w$ is the weighted agent which can be calculated from (1), and X^P is a matrix keeps the previous best found positions of all agents. ${}^{t}\mathbf{X}_j^P$ is the arbitrarily selected agent from the matrix of X^P with uniform probability. The weighted agent (X^W) is the weighted average of all available agents which is defined as follow:

$$\mathbf{X}^W = \sum_{i=1}^{M} \bar{c}_i^w \mathbf{X}_i^P \qquad (2)$$

$$\bar{c}_i^w = \left(\hat{c}_i^w \middle/ \sum_{i=1}^{M} \hat{c}_i^w\right)$$

$$\hat{c}_i^w = \frac{\max\limits_{1 \le k \le M}\left(f(\mathbf{X}_k^P)\right) - f(\mathbf{X}_i^P) + \varepsilon}{\max\limits_{1 \le k \le M}\left(f(\mathbf{X}_k^P)\right) - \min\limits_{1 \le k \le M}\left(f(\mathbf{X}_k^P)\right) + \varepsilon} \quad i = 1, 2, ..., M$$

$$(3)$$

where M is the number of all particles, $\mathbf{x}^w$ denotes the position vector of the weighted particle, $\hat{c}_i^w$ is the coefficient shows influence of each particle on the weighted particle, f (.) is the objective function of the optimization problem, $\max\limits_{1 \le k \le M}\left(f(\mathbf{X}_k^P)\right)$ and $\min\limits_{1 \le k \le M}\left(f(\mathbf{X}_k^P)\right)$, respectively, are the worst and best objective values among all particles, ε is a small positive number avoids division by zero.

ISA uses the improved fly-back (IFB) technique to handle any problem's constraints [29]. On contrary of the conventional penalty approach(s), the improved fly-back approach is an ad-hoc parameters free approach. This method is described in details in reference [1] however to give more illustrations this method is described concisely. The proposed IFB consists of three main steps [29]:

(i)   The problem's constraints are divided into two categories as characteristic (those need to objective function evaluation to check their violations) and numeric (those do not need to objective function evaluation to check their violations).

(ii) Each component of the current agent which violates any numeric constraints is replaced with corresponding components of the weighted agent.

(iii) If updated agent is feasible (does not violate the characteristic constraints), and provides better objective value it is hold and matrix of X^P is updated, but if not, it is rest to its prior best position stored in X^P.

3. Fuzzy adjusting the tendency factor

As described in the previous section the tendency factor (τ_i) determines the search pattern for each agent. In the standard version of ISA this factor is randomly generated, but to determine it more properly, it can be designated via a fuzzy

mechanism. Since the focused of the current work is not to addressed the definition of the fuzzy mechanism to prevent any prolongation the details about this fuzzy system is not provided. The interested readers can find this mechanism definition and work process in reference [32]. The membership function plays determinative role in the performance of the fuzzy system, thus the different types of these functions are assessed. In this regard, initially a definition of these membership functions are explained in following.

3.1. 3.1 Membership function

The membership function of a fuzzy set is a generalization form of the indicator function in the conventional sets. In the fuzzy logic, it represents the degree of truth as an extension of valuation. It should be noted that probabilities and degrees of truth are two different definitions and they are conceptually distinct. The fuzzy truth signifies membership in unclearly defined sets, but not likelihood of some conditions or events. Zadeh [32] introduced the membership functions for the first time in the literature. A fuzzy set $\tilde{A}$ in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as below:

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)) \quad y \in U\} \tag{4}$$

where $\mu_{\tilde{A}}(\cdot)$ = membership function of A while $\mu_{\tilde{A}}(\cdot) \in [0,1]$. The membership function $\mu_{\tilde{A}}(\cdot)$ maps U to the membership space M. Different types of membership functions involved in the current work are addressed in Fig. 1.

4. Numeric tests

This section is devoted to testing the effect of membership functions on the search behavior of the proposed ISA method. In this regard, two group of examples are selected. First, non-constrained mathematical functions and second, the constrained mechanical problems. The results are reported through illustrative tables.

4.1. Mathematical benchmark functions

In this section five well-known benchmark functions taken into consideration. These functions are shifted, while the fourth one is also rotated. These functions can change proposed optimization algorithm from different aspects. For instance, the first function has a smoother search space while the third one has a very noisy search domain. These functions are chosen in such a way that all membership functions be tested on both simple and complex search domains. The corresponding functions are schematically plotted and their formulations tabulated in Table 1. The dimension of these functions is taken as D=60 in the current work. Maximum number of iterations is set as 10000*D. To prevent any premature convergence, each problem runs for 30 times.

The results obtained from the numerical experiments are addressed in the Table 2. The successful rate (SR) and standard deviation (Std.) for all test functions are reported in this table. SR is defined as percentage of runs which algorithm can achieve the predefined accuracy level within the maximum allowable number of iterations. The accuracy level is set as 1E-2 for all functions in this investigation. Based on the results presented in Table 2, it can be worthy expressed that the types of membership function do not seriously affect the search performance of the algorithm. Therefore, the triangular MF seems to have top priority for implementation in such problems, since it is the simplest membership function which can provide the requirements.

4.2. Welded beam constrained problem

The welded beam problem shown in Fig. 2 is examined as the constrained optimization problem. The objective of this problem is the total cost of the welded beam which should be minimized involving seven different constraints based on the deflection (δ), shear stress (τ), normal stress (σ), etc. The welding's properties (l,h) and cross-section (b,t) specifications are taken into account as the design variables of the problem.
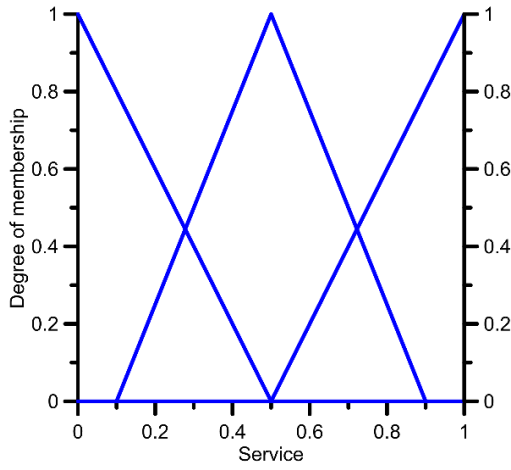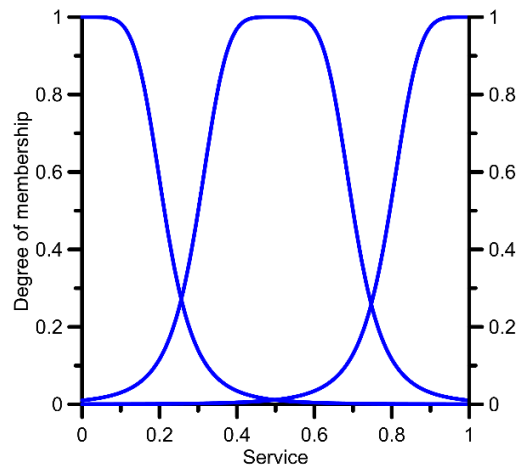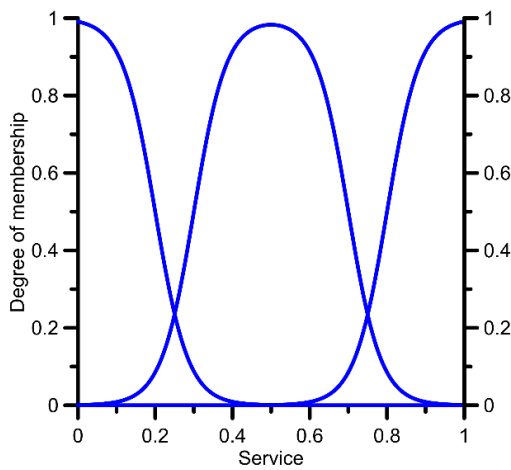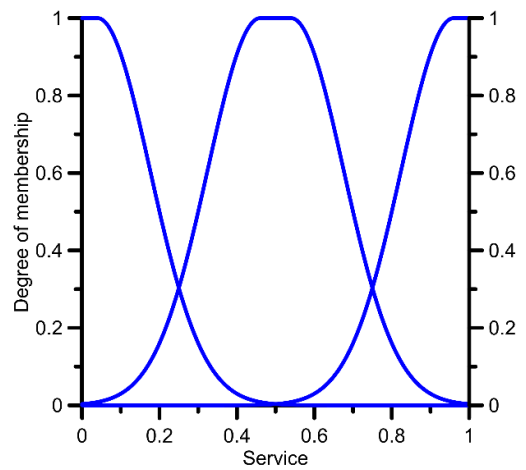
a. MF1: Triangular
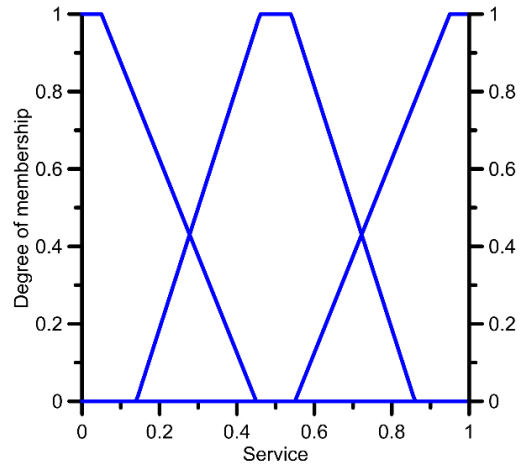
b. MF2: Trapezoid
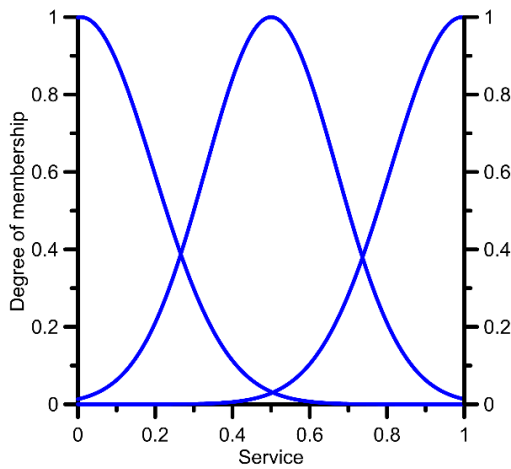
c. MF3: Gaussian

d. MF4: Gbellmf

e. MF5: Sigmoidal

f. MF6: Gaussian2

Fig. 1. Different types membership functions (MFs)
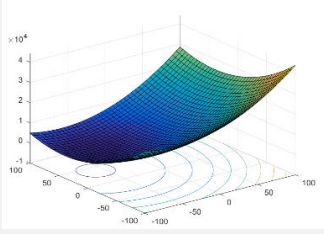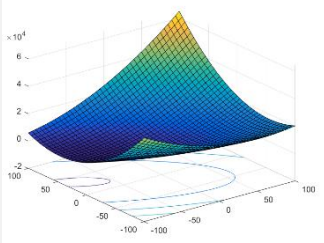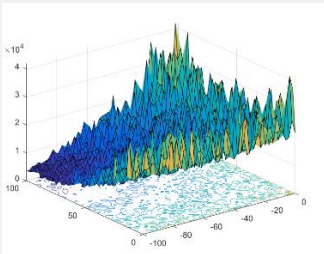
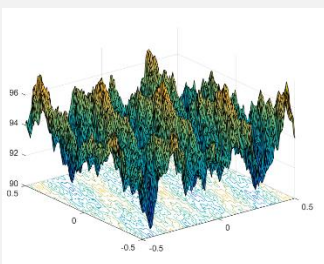**Table 1.** The benchmark functions properties
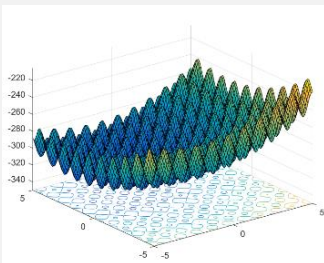
Function	Plot	Formulation	f_{bias}
F1: Shifted Sphere f1		$F_1(\mathbf{X}) = \sum_{i=1}^{D} z_i^2 + f\_bias,$ $\mathbf{Z} = \mathbf{X} - \mathbf{O}, \mathbf{X} = [x_1, x_2, ..., x_D]$ $\mathbf{X} \in [-100, 100]$	-450
F2: Shifted Schwefel's 1.2		$F_2(\mathbf{X}) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j^2 \right) + f\_bias,$ $\mathbf{Z} = \mathbf{x} - \mathbf{O}, \mathbf{x} = [x_1, x_2, ..., x_D]$ $\mathbf{x} \in [-100, 100]$	-450
F3: Shifted Schwefel's 1.2 with Noise in Fitness		$f(\mathbf{X}) = \left(\sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2 \right)(1 + 0.4\,|\,N(0,1)\,|) - 450$ $\mathbf{Z} = \mathbf{X} - \mathbf{o}, \quad \mathbf{X} = [x_1, x_2, ..., x_D], \quad \mathbf{X} \in [-100, 100]$	-450
F4: Shifted rotated Weierstrass		$F_4(\mathbf{X}) = \sum_{i=1}^{D} \left(\sum_{k=1}^{k_{max}} \left[a^k \cos\left(2\pi b^k (z_i + 0.5) \right) \right] \right)$ $-D \sum_{k=1}^{k_{max}} \left[a^k \cos\left(2\pi b^k (z_i + 0.5) \right) \right] + f\_bias,$ $a = 0.5, b = 3, k_{max} = 20,$ $\mathbf{z} = (\mathbf{x} - \mathbf{o}) \times \mathbf{M}, \mathbf{x} = [x_1, x_2, ..., x_D]$ $\mathbf{x} \in [-0.5, +0.5]$	90
F5: Shifted Rastrigin's		$F_5(\mathbf{X}) = \sum_{i=1}^{D} \left(z_i^2 - 10\cos(2\pi \mathbf{z}_i) + 10 \right) + f\_bias,$ $\mathbf{z} = (\mathbf{x} - \mathbf{o}) \times \mathbf{M}, \mathbf{x} = [x_1, x_2, ..., x_D]$ $\mathbf{x} \in [-5, +5]$	-330

Table 2. The results for different membership functions

| Test Functions		MF1	MF2	MF3	MF4	MF5	MF6
F1	Mean	-12.98	-12.93	-12.91	-12.97	-12.94	-12.98
	Std.	8.12E-8	8.01E-8	8.19E-8	8.08E-8	8.58E-8	8.41E-8
	SR	100	100	100	100	100	100
F2	Mean	12.81	12.88	12.82	12.80	12.82	12.86
	Std.	6.22E-8	6.87E-8	7.01E-8	6.55E-8	6.41E-8	6.66E-8
	SR	100	100	100	100	100	100
F3	Mean	-2.28	-2.35	-2.33	-2.33	-2.30	-2.29
	Std.	5.02E-3	5.51E-3	6.82E-3	5.77E-3	5.14E-3	5.59E-3
	SR	73	70	73	80	73	80
F4	Mean	2.21	2.23	2.20	2.22	2.22	2.24
	Std.	7.14E-1	7.11E-1	7.09E-1	7.88E-1	7.07E-1	7.62E-1
	SR	0	0	0	0	0	0
F5	Mean	2.01	2.01	2.05	2.12	2.09	2.08
	Std.	8.10E-1	8.08E-1	8.12E-1	8.11E-1	8.57E-1	8.02E-1
	SR	0	0	0	0	0	0
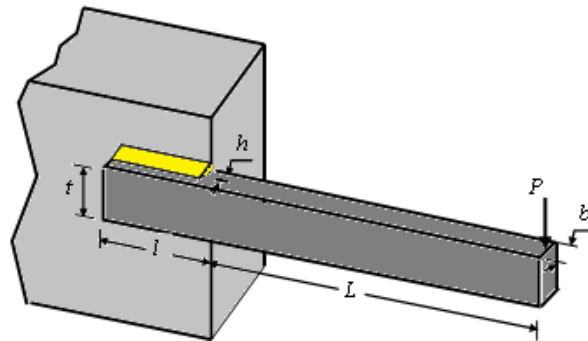
Fig. 2. The welded beam problem

$$\mathbf{X} = \{x_1, x_2, x_3, x_4\} \tag{5}$$

$$\text{cost}(\mathbf{X}) = 1.10471x_1^2 x_2 + 0.04811x_3 x_4(14 + x_2) \tag{6}$$

Subject to

$$g_1(x) = \tau(x) - \tau_{\max} \leq 0 \tag{7a}$$

$$g_2(x) = \sigma(x) - \sigma_{\max} \leq 0 \tag{7b}$$

$$g_3(x) = x_1 - x_4 \leq 0 \tag{7c}$$

$$g_4(x) = 0.10471x_1^2 + 0.04811x_3 x_4(14 + x_2) - 5 \leq 0 \tag{7d}$$

$$g_5(x) = 0.125 - x_1 \leq 0 \tag{7e}$$

$$g_6(x) = \delta(x) - \delta_{\max} \leq 0 \tag{7f}$$

$$g_7(x) = p - p_c(x) \leq 0 \tag{7g}$$

$$0.1 \leq x_1 \leq 2, \quad 0.1 \leq x_2 \leq 10, \quad 0.1 \leq x_3 \leq 10, \quad 0.1 \leq x_4 \leq 2$$

where

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \qquad (8a)$$

$$\tau' = \frac{P}{\sqrt{2}x_1 x_2} \quad \tau'' = \frac{MR}{J} \quad M = P\left(L + \frac{x_2}{2}\right) \qquad (8b)$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \qquad (8c)$$

$$J = 2\left\{\sqrt{2}x_1 x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \qquad (8d)$$

$$\sigma(x) = \frac{6PL}{x_4 x_3^2} \quad \delta(x) = \frac{4PL^3}{Ex_3^3 x_4} \qquad (8e)$$

$$P_c(x) = \frac{4.013\sqrt{E\left(x_3^2 x_4^6 / 36\right)}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \qquad (8f)$$

in which P = 6000 lb, L = 14 in., E = 30 x 10^6 psi, G = 12 x 10^6 psi, τ_{max} =13,600 psi, σ_{max} = 30,000 psi, and δ_{max} = 0.25 in.

The results obtained from the numerical investigations performed for this example with different membership functions is tabulated in Table 3. As can be seen from the results given in Table 3, variation of the membership function type has not a significant affect on the search capability of the algorithm.

4.3. A gear system

The gear system shown in Fig. 3 is evaluated as the last example in the current study. The objective of current example is to minimize the gear size ratio in the given gear mechanism of a train system. The objective function of this problem is mathematically illustrated as follows:

$$\text{Minimize } f(\mathbf{X}) = \left(\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}\right)^2$$

subject to: $\qquad\qquad\qquad\qquad\qquad\qquad (9)$

$$12 \le x_i \le 60 \quad i = 1, 2, 3, 4$$

where parameters of x_1, x_2, x_3 and x_4 give the number of gear teeth and range of them also $x_i \in [12, 60], i = 1, 2, 3, 4$. The proposed ratio is formulated as follows:

$$\text{The gear ratio} = \frac{\text{angular velocity of output shaft}}{\text{angular velocity of input shaf}} \quad (10)$$

The results obtained with utilizing different membership functions are presented in Table 4. Studying the results given in Table 4, it can be argued that similar to prior examples, the membership function type has no any particular effect in the search capability of the algorithm.
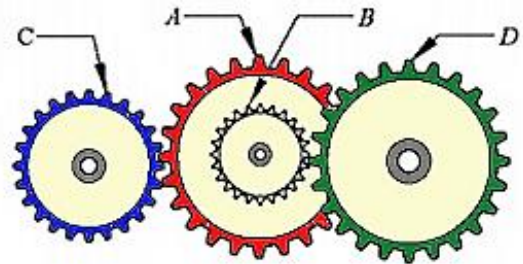

Fig 3. The gear system

Table 3. Welded beam example's results for different membership functions

Value	MF1	MF2	MF3	MF4	MF5	MF6
Mean	1.724856	1.724858	1.724856	1.724857	1.724856	1.724857
Std.	0.00	1.20E-7	0.00	1.04E-7	0.00	1.98E-8

Table 4. Gear system results

Value	MF1	MF2	MF3	MF4	MF5	MF6
Best	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12	2.70E-12
Std.	4.5E-12	4.51E-12	4.58E-12	4.51E-12	4.55E-12	4.56E-12

5. Conclusion

The current study deals with evaluating the effect of membership function types on the performance of the fuzzy-reinforced metaheuristic algorithms. In this regard, the Interactive Search Algorithm (ISA) is selected as the pilot optimization approach. The six prevalent types of membership functions named as triangular, trapezoidal, Gaussian, Sigmoidal and Bell are considered to adjust the fuzzy mechanism. It should be noted, among the all studied membership functions triangular membership function is the simplest one due to its linear approach. Five different none-constrained mathematical functions and two constrained mechanical problems are assessed as the benchmark problems. Achieved outcomes are summarized in the illustrative tables. The attained results show that types of membership function do not seriously affect the performance of the fuzzy module of the algorithm. Consequently, to define a fuzzy mechanism, picking the simplest membership function which can meet the requirements would be preferred.

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