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RESEARCH ARTICLE

Effect of Winkler foundation, inhomogeneity and orthotropy on the frequency of plates

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Abstract

This study investigates the frequencies of inhomogeneous orthotropic plates on elastic foundation. The inhomogeneity of orthotropic materials varies linearly with the coordinates of length and thickness. The basic equation is derived applying the Donnell-Mushtari theory and solved using the Galerkin method. The effects of inhomogeneity, Winkler elastic foundation and orthotropy on the frequencies are investigated in detail.

Keywords

Free vibration; Inhomogeneous orthotropic materials; Plates; Winkler elastic foundation

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1. Introduction

Inhomogeneous structures are widely used in aviation, aerodynamic structure, space vehicles, light-alloy structure of cars and in other engineering structures. The increase in the number of constructive variables extends the possibilities of advanced composite materials, as well as stability and vibration behaviors may be significantly altered. The reason for the appearance of heterogeneity of the material can be manufacturing technology, thermal and mechanical treatment, heterogeneity of compositions and a number of other reasons [1]. Significant contributions to consider different types of inhomogeneity are given in references [2-7]. In many practical applications, composite plates are in contact with soils or other solid particles and can have significant and unavoidable effect on their behaviors. A comprehensive review of elastic foundation models is discussed in the work by Gorbunov-Posadov et al. [8]. Vibration of homogeneous and non-homogeneous orthotropic plates resting on elastic foundation, which has practical applications in civil, mechanical, marine and aerospace engineers have been extensively studied by using various analytical and numerical methods [9-15].

In majority of the above-mentioned-studies, the elastic properties of inhomogeneous orthotropic materials were varied depending on the coordinate of the thickness or the coordinates in the plane, separately. The proposed study describes the free vibration of heterogeneous orthotropic plates on the Winkler elastic foundation, in which the inhomogeneity linearly varies along the length and thickness together.

2. Formulation of the problem

The configuration of rectangular inhomogeneous orthotropic plate with the length *a*, the width *b* and the thickness *h* resting on Winkler elastic

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foundation is illustrated in Fig. 1. The plate referred to a system of rectangular coordinate system *Oxyz*. The mid-plane being $z = 0$ and the origin is at one corners of the orthotropic plate. $z = 0$. The x and y axes are taken along the principle directions of orthotropy and z axis is normal to them.

It is assumed that Young's moduli and shear modulus of the heterogeneous orthotropic plate vary linearly with the coordinates x and z as

$$E_1 = E_1^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left[1+\mu_2\frac{z}{h}\right]$$
$$E_2 = E_2^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left[1+\mu_2\frac{z}{h}\right] \quad (1)$$
$$G_{12} = G_{12}^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left[1+\mu_2\frac{z}{h}\right]$$

while density ρ and Poisson's ratios ν_1 and ν_2 are assumed to be constant [2-7]. Here, E_1^0 and E_2^0 are the Young's moduli in x and y directions, G_{12}^0 is the shear modulus and ρ is the density of the homogeneous orthotropic plate. Here $X = x / a$ and $Z = z / h$ are the dimensionless variables in x and z directions; μ_1 and μ_2 are non-homogeneity parameters varied from zero to unity. As $\mu_1 = 1$ and $\mu_2 = 0$, the heterogeneous material is converted into a homogeneous material.

3. Basic relations

Based on the classical shell theory, the relationships between the stresses and strains at an arbitrary point of inhomogeneous orthotropic plates are written in the following form [2-7]:

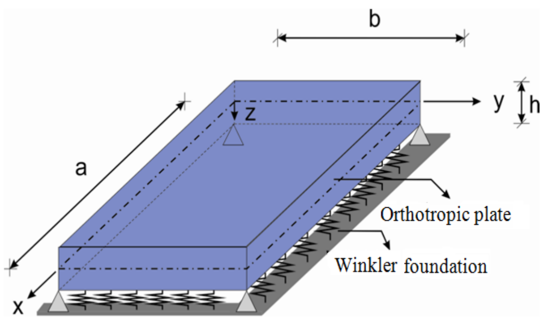

Fig. 1. The inhomogeneous orthotropic rectangular plate on the elastic foundation and the coordinate system

$$\sigma_{11} = \frac{1}{1-\nu_1\nu_2}E_1^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left[1+\mu_2\frac{z}{h}\right] \times\left[e_{11}+\nu_1 e_{22} - z\left(\frac{\partial^2 w}{\partial x^2}+\nu_1\frac{\partial^2 w}{\partial y^2}\right)\right]$$
$$\sigma_{22} = \frac{1}{1-\nu_1\nu_2}E_2^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left[1+\mu_2\frac{z}{h}\right] \times\left[e_{22}+\nu_2 e_{11} - z\left(\frac{\partial^2 w}{\partial y^2}+\nu_2\frac{\partial^2 w}{\partial x^2}\right)\right] \quad (2)$$
$$\sigma_{12} = G_{12}^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left[1+\mu_2\frac{z}{h}\right] \times\left(e_{12} - 2z\frac{\partial^2 w}{\partial x\partial y}\right)$$

where w is the small displacement in z direction and $e_{ij}(i,j=1,2)$ are the strains in the middle plane of inhomogeneous orthotropic plate.

The force and moment resultants are expressed by the following relations [15, 16]:

$$(T_{11},T_{22},T_{12},M_{11},M_{22},M_{12}) = \int_{-h/2}^{h/2}(\sigma_{11},\sigma_{22},\sigma_{12},\ z\sigma_{11},\ z\sigma_{22},\ z\sigma_{12})dz \quad (3)$$

Since there are no external forces in the plane of plate $(T_{11}=T_{22}=T_{12}=0)$, it is therefore assumed that the resultant forces are everywhere is equal to zero. In this case, the following conditions can be written:

$$A_1(e_{11}+\nu_1 e_{22}) - A_2(\chi_{11}+\nu_1\chi_{22}) = 0$$
$$A_1(e_{22}+\nu_2 e_{11}) - A_2(\chi_{22}+\nu_2\chi_{11}) = 0 \quad (4)$$
$$A_1 e_{12} - A_2\chi_{12} = 0$$

where $A_1 = h,\ A_2 = \mu_2 h^2/12$ and $\chi_{ij}(i,j=1,2)$ are the curvatures of the deformed plate.

Taking into account Eq. (2) in the expression (3), after integrating, we obtain the following expressions for the moments:

$$M_{11} = -\left(1-\frac{\mu_2^2}{12}\right)D_1^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left(\frac{\partial^2 w}{\partial x^2}+\nu_1\frac{\partial^2 w}{\partial y^2}\right)$$
$$M_{22} = -\left(1-\frac{\mu_2^2}{12}\right)D_2^0\left[1+\frac{x}{a}(\mu_1-1)\right]\left(\frac{\partial^2 w}{\partial y^2}+\nu_2\frac{\partial^2 w}{\partial x^2}\right)$$
$$M_{12} = -\left(1-\frac{\mu_2^2}{12}\right)D_{12}^0\left[1+\frac{x}{a}(\mu_1-1)\right]\frac{\partial^2 w}{\partial x\partial y} \quad (5)$$

where D_1^0, D_2^0, D_{12}^0 are flexural rigidities of the homogeneous orthotropic plate and are defined as:

$$D_1^0 = \frac{E_1^0 h^3}{12(1-\nu_1\nu_2)},\ D_2^0 = \frac{E_2^0 h^3}{12(1-\nu_1\nu_2)},\ D_{12}^0 = \frac{G_{12}^0 h^3}{12} \tag{6}$$

The partial differential equation of the motion of orthotropic plate resting on the Winkler elastic foundation can be written as [15]:

$$\frac{\partial^2 M_{11}}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_{22}}{\partial y^2} - Kw - \rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{7}$$

where K (in N / m^3) is the modulus of subgrade reaction for the Winkler elastic foundation [15,16].

Substituting Eq. (5) into Eq. (7), after elementary transformations, we obtain the equation of motion of inhomogeneous orthotropic plate resting on the Winkler elastic foundation as:

$$\begin{aligned} &\left[1+\frac{x}{a}(\mu_1-1)\right]\times\left[D_1^0\frac{\partial^4 w}{\partial x^4}+D_2^0\frac{\partial^4 w}{\partial y^4}\right.\\ &\left.+\left(D_1^0\nu_2+\nu_1 D_2^0+4D_{12}^0\right)\frac{\partial^4 w}{\partial x^2\partial y^2}\right]\\ &+2D_1^0\frac{\mu_1-1}{a}\left(\frac{\partial^3 w}{\partial x^3}+\nu_1\frac{\partial^3 w}{\partial y^2\partial x}\right)+4D_{12}^0\frac{\mu_1-1}{a}\frac{\partial^3 w}{\partial x\partial y^2}\\ &+\frac{12}{12-\mu_2^2}Kw+\frac{12}{12-\mu_2^2}\rho h\frac{\partial^2 w}{\partial t^2}=0 \end{aligned} \tag{8}$$

4. The solution of equation of motion

We assume that the boundary conditions for the bending of continuous inhomogeneous orthotropic plates coincide with the usual ones in the homogeneous isotropic plate.

We take the harmonic solution of Eq. (8) in the form [15,16]

$$w(x,y,t) = A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{i\omega t} \tag{9}$$

which satisfies the simply-supported boundary conditions edges of the inhomogeneous orthotropic rectangular plates, here $i=\sqrt{-1}$, m and n are wave numbers, and A is the unknown amplitude.

Substituting Eq. (9) into Eq. (8), then applying the Galerkin's method to the resulting equation for $m = n = 1$, the expression for the frequency of free vibration of inhomogeneous orthotropic plates resting on the Winkler elastic foundation is obtained as:

$$\begin{aligned} \omega = &\left(\frac{K_w}{\rho h} + \frac{(12-\mu_2^2)}{12}\frac{(1+\mu_1)}{2\rho h}\right.\\ &\left.\times\left[\eta_1^4 D_1^0 + \eta_2^4 D_2^0 + \eta_1^2\eta_2^2\left(D_1^0\nu_2 + D_2^0\nu_1 + 4D_{12}^0\right)\right]\right)^{1/2} \end{aligned} \tag{10}$$

where $\eta_1 = \pi / a$ and $\eta_2 = \pi / b$.

The values of the dimensionless frequency parameter for inhomogeneous orthotropic plates resting on the Winkler elastic foundation is found as:

$$\omega_1 = \omega\frac{a^2}{h}\sqrt{\rho / E_2^0} \tag{11}$$

As $\mu_1 = 1, \mu_2 = 0$ and $K_w = 0$, the frequency coincides with the frequency of the homogeneous orthotropic plate without an elastic foundation and can be expressed as:

$$\omega_0 = \sqrt{\frac{\eta_1^4 D_1^0 + \eta_2^4 D_2^0 + \eta_1^2\eta_2^2\left(D_1^0\nu_2 + D_2^0\nu_1 + 4D_{12}^0\right)}{\rho h}} \tag{12}$$

5. Results and discussion

Table 1 presents a comparative study of the dimensionless frequency parameters ω_1 for the flexural modes of thin homogeneous isotropic square plates on Winkler foundation. Dimensionless parameters are specified as

$$\omega_1 = \frac{\omega b^2}{\pi^2}\sqrt{\rho h / D^0},\ \bar{K} = \frac{Ka^4}{D^0},\ \nu_0 = 0.3,\ \rho = 1 \tag{13}$$

Table 1. Comparison of frequency parameters for the flexural modes of thin square plates on homogeneous Winkler elastic foundation ($a/b=1, b/h=100$)

$\bar{K}$	Reference	ω_1
10^2	Leissa [15]	2.2420
	Zhou et al. [10]	2.2413
	Present study	2.2420
5×10^2	Leissa [15]	3.0221
	Zhou et al. [10]	3.0214
	Present study	3.0220

Table 2. Variation of frequency parameters of the inhomogeneous orthotropic rectangular plates on inhomogeneous viscoelastic and elastic foundations versus E_1 / E_2 ($a/b = 0.5$, $b = 1$m, $a/h = 50$)

E_1 / E_2	$\mu_1 = 1$ $\mu_2 = 0$	$\mu_1 = 2$ $\mu_2 = 0$	$\mu_1 = 1$ $\mu_2 = 1$	$\mu_1 = 2$ $\mu_2 = 1$
$K_w^0 = 0$				
10	3.084	3.777	2.953	3.616
25	2.995	3.668	2.868	3.512
40	2.964	3.630	2.838	3.476
55	2.947	3.609	2.821	3.455
$K_w^0 = 5 \times 10^6 (\text{N/m}^3)$				
10	3.320	3.972	3.198	3.819
25	3.238	3.869	3.120	3.721
40	3.209	3.833	3.093	3.687
55	3.193	3.813	3.077	3.668

Table 3. Variation of frequency parameters of the inhomogeneous orthotropic rectangular plates on Winkler elastic foundation a/b ($a/h = 50$, $b = 1$m)

a/b	$\mu_1 = 1$ $\mu_2 = 0$	$\mu_1 = 2$ $\mu_2 = 0$	$\mu_1 = 1$ $\mu_2 = 1$	$\mu_1 = 2$ $\mu_2 = 1$
$K_w^0 = 0$				
0.5	2.931	3.590	2.806	3.437
1.0	3.237	3.964	3.099	3.795
1.5	3.957	4.847	3.789	4.640
2.0	5.216	6.388	4.994	6.116
$K_w^0 = 5 \times 10^6 (\text{N/m}^3)$				
0.5	3.178	3.795	3.064	3.650
1.0	3.674	4.328	3.553	4.174
1.5	4.493	5.293	4.346	5.105
2.0	5.766	6.844	5.566	6.591

These values are taken from the study of Leissa [15]. As seen, the natural frequencies are in good agreement with the results of Leissa [15] and Zhou et al. [10] for thinner plates.

Variation of the dimensionless frequency parameters for homogeneous and inhomogeneous rectangular plates with and without Winkler elastic foundation against E_1 / E_2 ratio for *a* / *b* = 0.5, *b* = 1m and *a* / *h* = 0.5 are presented in Table 2. The material properties of orthotropic plate are $E_1 / E_2 =$ 10, 25, 40 and 50, $E_2 = 206.9GPa$, $G_{12} = 0.5E_2$, $\nu_1 = 0.25$, $\rho = 1\text{kg/m}^3$. As Table 2 shows, the dimensionless frequency parameters for the considered cases are weakly decreased when E_1 / E_2 ratio increases.

Since the soil effect is not taken into account, the heterogeneity effect is approximately independent of E_1 / E_2 ratio, although the heterogeneity effect is considerable. For example, if the foundation effect is not taken into account, the effect of non-homogeneity is significant (22.47%), when the Young's and shear moduli change only in *x* direction, whereas, this effect is found to be -4.3%, when the Young moduli change only in *z* direction and the effect of heterogeneity is %17.3, as the Young's moduli change in *x* and *z* directions together. Here, the negative sign indicates that the value for the inhomogeneous plate is smaller than the homogeneous plate. Considering the foundation effect, significant changes occur in the effects of non-homogeneity on the frequencies. For example, as the Winkler foundation coefficient is taken to be $K_w = 5 \times 10^6$ N/m^3, the effect of non-homogeneity on the frequencies increases by up to 29%, as the Young's and shear moduli change only in *x* direction; this effect increases from 3.7% to 4.41%, when the Young and shear moduli vary only in *z* direction, the effect of non-homogeneity a weak increase from 23.83% to 24.47%, as the Young and shear moduli vary in *x* and *z* directions together, as E_1 / E_2 ratio increases from 10 to 55.

Variations of the dimensionless frequency parameters of homogeneous and inhomogeneous rectangular plates with and without Winkler elastic foundation against *a* / *b* ratio for *a* / *h* = 50 and *b* = 1m are presented in Table 3. The material properties of orthotropic plate are E_1^0 = 206.9GPa, E_2^0 = 20.69GPa, G_{12}^0 = 6.9GPa and ρ^0 = 1950kg/m^3 Table 3 shows that the values ω_1 increase with increasing a/b. The effect of the Winkler foundation on the frequency values first increases and then decreases, as the a/b increases from 0.5 to 2. For example, the effect of Winkler foundation on the frequencies of homogeneous orthotropic plates chances between 8.43% and 13.55%. For the inhomogeneous orthotropic plates, the effect of elastic foundation changes in the range of 29.48% and 33.76%, as the Young's and shear moduli change only in x direction. This effect changes in the range of 4.54% and 9.76%, when the Young's and shear moduli vary only in the z direction, and the range of the influence of the inhomogeneity is 24.53% to 29.01%, as the Young and shear moduli vary in the x and z directions together, since the ratio a/b increases from 0.5 to 2.

6. Conclusions

The purpose of this study is to investigate the effects of the inhomogeneous and elastic foundation on the frequency of orthotropic plates. The inhomogeneity of orthotropic materials varies linearly with the coordinates of the length and thickness. The basic equation is derived applying the Donnell-Mushtari theory and solved using the Galerkin's method. The effects of inhomogeneity, Winkler elastic foundation and orthotropy on the frequencies are investigated in detail.

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