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RESEARCH ARTICLE

Solution of housing contractor profit maximization problem

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Abstract

In this study, a new problem that aims to maximize the profit of housing contractors based on the limitations of the contractors is proposed. The budget and the available workforce of the contractor as well as market conditions and the existence of suitable locations can be given as examples of the limitations of the contractors. Formulation of the problem requires estimating the expected costs and the expected profits of the construction alternatives. The limitations on the budget of the contractor and the suitability of the subcontractors may prevent the execution of all the construction opportunities. This situation leads to the problem of selecting the most appropriate housing construction alternatives. The construction opportunities can be executed or not. This forms binary decision variables, and the problem can be solved by simplex or branch-and-bound methods. An additional construction alternative doubles the number of construction combinations that constitute the search domain of the problem. The number of construction opportunities is not expected to be excessive. For this reason, the problem can also be solved by complete enumeration. Three hypothetical housing contractor profit maximization problems are formulated and solved to measure both the computational demand and the ease of application of the simplex, branch-and-bound, and complete enumeration techniques. The comparison revealed that all the techniques provide the optimum result for the problem. Complete enumeration is the easiest technique to implement because of the small search domain of the problem.

1. Introduction

Construction investments require important amount of capital and workforce. Therefore, satisfactory profits are expected in order to commence the construction of a facility. Development of information technologies in construction sector ease the cost estimation and scheduling computations and reduce the cost of the estimation phase. The quantity take-off of construction items can be computed effortlessly and accurately by developed applications [1] or Building Information Modeling software [2, 3]. In addition to this artificial intelligence tools such as artificial neural network models are used to estimate the price of the housing at certain regions [4]. Therefore, the contractors can consider many housing construction opportunities by spending small amount of time and effort.

Urban renewal projects provide important opportunities for the housing contractors [5]. On the other hand, urban renewal projects are very risky projects [6]. In addition to this, limitations on the resources of the contractors such as budget and

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labor increase the risks of the housing projects. Many small and medium scale housing contractors do not utilize Enterprise Resource Planning systems or software [7]. Therefore their capabilities on the management of their resources are limited. The competition among the housing contractors is very tough that land owners' share is sometimes higher than the contractors' share. Construction sector requires stable economy to improve its competitiveness [8]. On the other hand, recently the contractors are affected seriously because of the economic situation and international relationships. Therefore, in order to protect their competitiveness effective decision support tools should be utilized during the decision making phase. This study aims to develop an assisting tool for the contractors for selecting the most suitable housing construction opportunities to maximize their profit. The restrictions on the budget and the resources are defined via additional variables. The most suitable construction opportunities are determined by simplex algorithm, branch-and-bound algorithm and complete enumeration. Implementation of the algorithms is explained in the methodology part, and the solution of the problem is presented in the case study part and the findings of the study are discussed in the last part.

2. Methodology

The housing contractor profit maximization problem is explained and the implementations of simplex, branch-and-bound and complete enumeration alternatives are briefly explained in the methodology part.

2.1. Definition of the problem

The examined problem aims to maximize the profit of the housing contractor who can execute at most n housing construction projects among the m housing construction opportunities. The objective function is expressed in Eq. 1.

$$\max f(X) = \sum_{i=1}^{m} p_i x_i \tag{1}$$

where x_i is the choice on the i^{th} housing construction alternative which can be construct or do not construct, p_i is the expected profit from the i^{th} construction alternative. The decision variable x_i can be 0 or 1 depending on the decision and it is expressed as in Eq. 2.

$$x_i \in \{0, 1\} \tag{2}$$

The limitation of construction of at most n facility is expressed as in Eq. 3.

$$\sum_{i=1}^{m} x_i \leq n \tag{3}$$

The limitation on the budget of the contractor is expressed in Eq. 4.

$$\sum_{i=1}^{m} \left(C_i x_i\right) \leq BD \tag{4}$$

In Eq. 4 C_i is the construction cost of the i^{th} facility and BD is the total budget of the contractor. The objective function is maximized without violating the restrictions. The maximum value of the Eq. 1 obtained without violating the restrictions is the maximum obtainable profit of the contractor.

2.2. Solution by simplex algorithm

The problem is transformed to simplex form. The maximization of the objective function is transformed to minimization of the objective function by multiplying it by -1. Slack variables are defined for each constraint in order to convert the inequality expressions to equations. Solution starts with the slack variables. The decision variable with the highest profit/cost ratio is substituted with the corresponding slack variable. The simplex table is updated if a new design variable is introduced. The opportunity of improving the solution can be monitored by the coefficients of the excluded variables. The solution process stops if there is not any possibility of improving the value of the objective function [9].

The construction opportunities are sorted descending with respect to their profit per cost ratio. The construction opportunity with the highest profit per cost ratio is included to the design variable set. If the restrictions are not violated, the next construction alternative is examined. If the examined activity requires execution of another alternative which is not currently in the design variable set, the considered activity is skipped. If the conditional alternative is included then the ignored activity is re-considered for possible inclusion to the design variable set. The sequential search of the variables continues while the restrictions on the budget and the number of housing construction are not violated. The obtained final solution is reported as optimum or near-optimum solution.

2.3. Solution by branch and bound algorithm

The branch-and-bound is a suitable algorithm for pruning out the improper solutions. This property avoids the local minimum of the search space and provides better solution. The problem is handled as a tree like structure as illustrated in Fig. 1. Each decision variable is considered at a separate level and at each level solution alternatives are branched. The solution strategy is to detect and eliminate infeasible branches to narrow the search domain. The decision variables are binary. Therefore, branches of the nodes contains the zero and one opportunities for the construction alternatives [10].

The housing construction alternatives are sorted descending according to their profit to cost ratio. The initial solution contains the construction opportunities with the highest profit to cost ratio. If the restriction on the budget does not allow entrance of the construction opportunities, the restriction is relaxed and the first node is branched on the relaxed restriction. The alternative with the highest profit per cost which is not included to the design variable set is considered for the inclusion to the variable set. Among the included activities, the activity with the lowest profit per cost ratio is considered for possible removal from the design variable set. The search continues until the upper bounds of the examined nodes are less than the obtained current best solution.

2.4. Solution by complete enumeration

The examined problem can have m construction alternatives which can be only 0 or 1. Each construction alternative has 2 possible decisions and m construction alternative ends up with 2^m combinations. All of the combinations may not be feasible since some of them might violate the restrictions. Complete enumeration does not execute a sophisticated search and also visit the infeasible solutions. Infeasible solutions violate the restrictions and can provide higher objective function value than feasible solutions. In order to prevent this situation penalty values are assigned for each violated restriction. The modified objective function is given in Eq. 5.

$$\max f(X) = \sum_{i=1}^{m} p_i x_i + \sum_{i=1}^{m} \alpha_i O_i \tag{5}$$

$$\alpha_i \in \{0, 1\} \tag{6}$$

In Eq. 5 and 6, α_i is the state of the restriction which is 0 if the restriction is satisfied and 1 if the restriction is violated. O is the predefined negative penalty value for the violation of the restrictions and r is the number of restrictions. The values of the penalty functions are determined in order to ensure the elimination of infeasible solutions. All of the combinations are evaluated according the Eq. 5 and the obtained maximum objective function is recorded.

In this study, instead of implicit enumeration, complete enumeration is implemented.

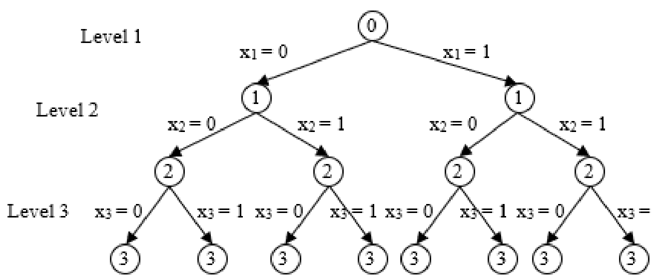


Fig. 1. Implementation of branch-and-bound algorithm for binary decision variables

In implicit enumeration only the feasible solutions are evaluated and the infeasible solutions are eliminated. The execution of the implicit enumeration requires formation of nested if statements to detect the existence of any violation. The variables are certain to be non-negative, but the remaining restrictions should be checked by the nested if statements. Therefore, the number of nested if statements would be equal to the number of the restrictions of the problem. In order to simplify the code written for the evaluation of the whole search domain, penalty functions are implemented for the infeasible solutions. In this case the violations of the restrictions are queried by sole if statements. The penalty functions reduce the value of the objective function considerably and infeasible solutions have lower values than the feasible solutions. The *max* and *index* functions of the spreadsheet software find the best feasible solution.

3. Case Studies

In this study three contractor profit maximization problems are defined and solved by simplex, branch-and-bound and complete enumeration algorithms. The problems and their solutions are illustrated in this part.

3.1. Case problem 1

A contractor considers executing housing projects in Çöşnük, Fahri Kayhan, Bostanbaşı and Sıtmapınarı districts. The costs of these constructions will be 5, 8, 7 and 6 million $ respectively. The expected profits of the mentioned projects are 1, 1.5, 1.25 and 1.1 million $ respectively. If the contractor's budget is 18 million $ and can execute at most three constructions what can be the highest profit?

Variables of the problem are represented as; x_1 Construct in Çöşnük or not (1 or 0), x_2 Construct in Fahri Kayhan or not (1 or 0), x_3 Construct in Bostanbaşı or not (1 or 0), x_4 Construct in Sıtmapınarı or not (1 or 0). The problem is formulated as below.

$$\max F(X) = 1x_1 + 1.5x_2 + 1.25x_3 + 1.1x_4 \quad \text{(objective function)}$$

$$5x_1 + 8x_2 + 7x_3 + 6x_4 \leq 18 \quad \text{(restriction on the budget)}$$

$$x_1 + x_2 + x_3 + x_4 \leq 3 \quad \text{(restriction on the number of constr.)}$$

$$x_1 \in \{0, 1\} \quad \text{(decision variable 1)}$$

$$x_2 \in \{0, 1\} \quad \text{(decision variable 2)}$$

$$x_3 \in \{0, 1\} \quad \text{(decision variable 3)}$$

$$x_4 \in \{0, 1\} \quad \text{(decision variable 4)}$$

The variables and the restriction equations are normalized as following.

$$\min f(X) = -1x_1 - 1.5x_2 - 1.25x_3 - 1.1x_4$$

$$\min f(X) = -1x_1 - 1.5x_2 - 1.25x_3 - 1.1x_4 - f$$

$$5x_1 + 8x_2 + 7x_3 + 6x_4 + x_5 = 18$$

$$x_1 + x_2 + x_3 + x_4 + x_6 = 3$$

$$x_1 + x_7 = 1$$

$$x_2 + x_8 = 1$$

$$x_3 + x_9 = 1$$

$$x_4 + x_{10} = 1 \qquad x_j \geq 0, \; j = \{1,...,10\}$$

The profit per cost ratios of the construction alternatives are given in Table 1.

3.1.1. Solution of case problem 1 by simplex algorithm

The solution begins with the starting point $x_1 = x_2 = x_3 = x_4 = 0$, this states that the available budget is $x_5 = 18$ and the available construction opportunities is $x_6 = 3$. The slack variables are $x_7 = x_8 = x_9 = x_{10} = 1$. The profit is $-f = 0$. This situation is represented in Table 2.

The activity x_1 has the highest profit per cost ratio and it is inserted to the design variable set. Maximum profit increase per unit cost is provided by including x_1 and removing x_7. The updated situation is represented in Table 3. The values of the variables are $x_2 = x_3 = x_4 = 0$, $x_1 = 1$, $x_5 = 13$, $x_6 = 2$, $x_7 = x_9 = x_{10} = 1$, $x_8 = 0$, $-f = 1$. The parameter with the second highest profit per cost is x_2. It is inserted by removing x_8.

Table 1. Profit per cost ratios of the housing construction alternatives.

	x_1	x_2	x_3	x_4
Profit	1	1.5	1.25	1.1
Cost	5	8	7	6
P/C	0.200	0.188	0.179	0.183

Table 2. Initial situation of the first case study problem.

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	-f	b_i''	$\frac{b_i}{a_{ij}}$
x_5	5	8	7	6	1	0	0	0	0	0	0	18	18/5
x_6	1	1	1	1	0	1	0	0	0	0	0	3	3/1
x_7	1	0	0	0	0	0	1	0	0	0	0	1	1
x_8	0	1	0	0	0	0	0	1	0	0	0	1	X
x_9	0	0	1	0	0	0	0	0	1	0	0	1	X
x_{10}	0	0	0	1	0	0	0	0	0	1	0	1	X
-f	-1	-1.5	-1.25	-1.1	0	0	0	0	0	0	1	0	

Table 3. First variable is inserted for the first case study problem

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	-f	b_i''	$\frac{b_i}{a_{ij}}$
x_5	0	8	7	6	1	0	-5	0	0	0	0	13	13/8
x_6	0	1	1	1	0	1	-1	0	0	0	0	2	2/1
x_1	1	0	0	0	0	0	1	0	0	0	0	1	X
x_8	0	1	0	0	0	0	0	1	0	0	0	1	1/1
x_9	0	0	1	0	0	0	0	0	1	0	0	1	X
x_{10}	0	0	0	1	0	0	0	0	0	1	0	1	X
-f	0	-1.5	-1.25	-1.1	0	0	1	0	0	0	1	1	1

Obtained situation is represented in Table 4 where $x_3 = x_4 = 0$, $x_1 = 1$, $x_2 = 1$, $x_5 = 5$, $x_6 = 1$, $x_9 = x_{10} = 1$, $x_8 = 0$, $x_7 = 0$, $-f = 2.5$.

Table 4 represents that the remaining housing construction alternatives cannot be implemented because of the restriction on the budget. The red colored construction costs violate the budget. However, 5 million $ remaining budget indicate that there might be a better solution. Therefore, variable x_2 is removed and the not included alternative with the maximum cost per profit ratio, x_4 is entered to the design set. Table 3 can be used to check the state of the restrictions.

In Table 5 values of the variables are $x_2 = x_3 = 0$, $x_1 = x_4 = 1$, $x_5 = 7$, $x_6 = 1$, $x_8 = x_9 = 1$, $x_7 = x_{10} = 0$, $-f = 2.1$. In Table 5 insertion of variable x_3 is also checked at the last column and it is seen that x_3 can also be inserted to the design variable set.

The variable x_3 is also inserted to the design variable set and the obtained solution is shown in Table 6. The values of the parameters are listed as $x_1 = x_3 = x_4 = 1$, $x_2 = 0$ $x_5 = 0$, $x_6 = 0$, $x_8 = 1$, $x_7 = x_9 = x_{10} = 0$, $-f = 3.35$. Obtained solution cannot be improved because the housing construction alternatives with the highest profit per cost are implemented where possible.

3.1.2. Solution of case problem 1 by branch-and-bound algorithm

The variables are sorted according to their profit to cost ratios as x_1, x_2, x_4, and x_3. The initial solution is obtained by executing x_1 by spending 5 million $ and 13 million $ budget remains available. Decision variable x_2 is executed by spending 8 million $, and 5 million $ available budget remains. The variable x_4 cannot be executed therefore the constraints are relaxed and it is executed 5/6 = 0.833 pieces.

Table 4. Second variable is inserted for the first case study problem

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	-f	b_i''	$\frac{b_i}{a_{ij}}$
x_5	0	0	7	6	1	0	-5	-8	0	0	0	5	5/7 5/6
x_6	0	0	1	1	0	1	-1	-1	0	0	0	1	1/1
x_1	1	0	0	0	0	0	1	0	0	0	0	1	1/1
x_2	0	1	0	0	0	0	0	1	0	0	0	1	X
x_9	0	0	1	0	0	0	0	0	1	0	0	1	X
x_{10}	0	0	0	1	0	0	0	0	0	1	0	1	X
-f	0	0	-1.25	-1.1	0	0	1	1.5	0	0	1	2.5	

Table 5. Selection of design variable x_4 instead of x_2

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	-f	b_i''	$\frac{b_i}{a_{ij}}$
x_5	0	8	7	0	1	0	-5	0	0	-6	0	7	=7/7
x_6	0	1	1	0	0	1	-1	0	0	-1	0	1	1/1
x_1	1	0	0	0	0	0	1	0	0	0	0	1	X
x_8	0	1	0	0	0	0	0	1	0	0	0	1	X
x_9	0	0	1	0	0	0	0	0	1	0	0	1	1/1
x_4	0	0	0	1	0	0	0	0	0	1	0	1	X
-f	0	-1.5	-1.25	0	0	0	1	0	0	1.1	1	2.1	

Table 6. Optimum solution of the first case study problem

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	-f	b_i''	$\frac{b_i}{a_{ij}}$
x_5	0	8	0	0	1	0	-5	0	-7	-6	0	0	X
x_6	0	1	0	0	0	1	-1	0	-1	-1	0	0	X
x_1	1	0	0	0	0	0	1	0	0	0	0	1	X
x_8	0	1	0	0	0	0	0	1	0	0	0	1	X
x_3	0	0	1	0	0	0	0	0	1	0	0	1	X
x_4	0	0	0	1	0	0	0	0	0	1	0	1	X
-f	0	-1.5	0	0	0	0	1	0	1.25	1.1	1	3.35	

The initial starting point becomes X= [1; 1; 0; 0.833] with 1*1 + 1* 1.5+ 0.833*1.1 = 3.542 million $ profit. The relaxed solution provides the upper bound. The lower bound is obtained by rounding down the fractional part. The solution is not feasible therefore, the node is branched and the variable x_4 is made feasible by assigning 1 or 0 for it. The initial branch of the first case problem is shown in Fig. 2.

Solution attempt with $x_4 = 0$ leads to the execution of x_1 and x_2 with available 5 million $ budget. The relaxation of the restrictions provide execution of 5/7 = 0.714 pieces of x_3 which provides 3.393 million $ profit. The examination of x_3 is executed by branching the Node 1 which is illustrated in Fig. 3.

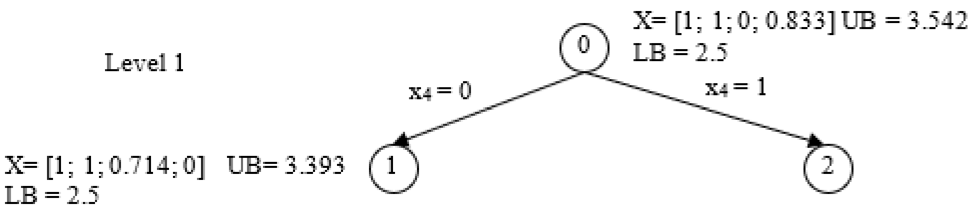


Fig. 2. Initial branch of the first case problem

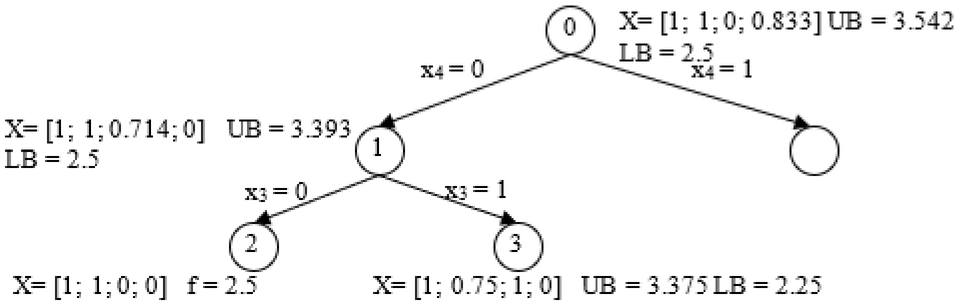


Fig. 3. Second branch of the first case problem

Node 2 does not violate any constraint and provides a feasible solution. The solution is a local maximum which executes x_1 and x_2 with 2.5 million $ profit. In node 3 execution of x_3 is examined. This violates the budget constraints therefore the integer constraint on the alternative x_2 is relaxed. Node 3 given in Figure 3 involves execution of x_2 0.75 pieces with 3.375 million $ profit. The profit is higher than the local maximum therefore the node is investigated. The branching of the node 3 is shown in Fig. 4.

Nodes 4 and 5 are formed by the branches of the node 3. Node 4 does not execute x_2 and executes x_1 and x_3 together. The objective function becomes 2.25 million $ which is worse than the present local maximum. Node 5 executes x_2 and excludes x_1 because of the restrictions on the budget. The obtained profit becomes 2.75 million $ which becomes the current best solution. This situation investigates the all possible branches related with the $x_4 = 0$ branch. Then $x_4 = 1$ branch is investigated by entering Node 6 as shown in Fig. 5.

Node 6 relaxes the restriction on the x_4 and executes the x_2 7/8 piece. This relaxation ends up with 3.4125 million $ profit which is the upper bound of the node. The upper bound is more than the current best solution. Therefore the Node 6 is investigated by forming nodes 7 and 8 as shown in Fig. 6.

Node 7 executes x_2 and x_4 which spends 14 million $ and 4 million $ remains. In this situation the remaining x_1 and x_3 alternatives are not possible to be executed. Therefore the Node 7 is the final node. The objective function is 2.6 million $ in Node 7 which is worse than the current best solution. Node 8 executes x_4 and does not execute x_2. This situation leaves 12 million $ available budget which permits execution of both x_1 and x_3. This node does not violate any restrictions and there is no need to investigate further branches. The obtained objective function is $1*1 + 1*1.25 + 1*1.1 = 3.35$ million $ profit which is the current best. There is not any remaining branch therefore; the obtained solution is the optimum solution of the problem

3.1.3. Solution of case problem 1 by complete enumeration

In order to execute complete enumeration process a spreadsheet application is prepared. The four design variable makes 16 housing construction alternatives. In order to form the decision alternatives of the search domain, the equation given in Eq. 7 is implemented for each x_i where $i = \{1,..,4\}$.

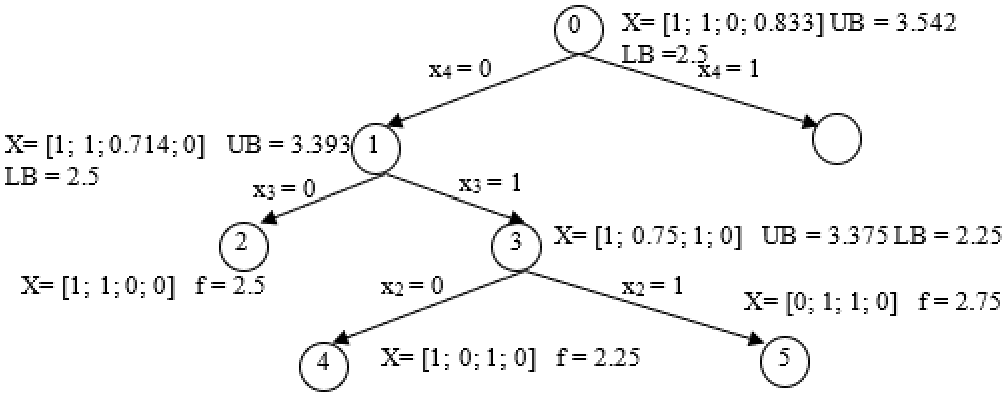


Fig. 4. Investigation of the branches of Node 3

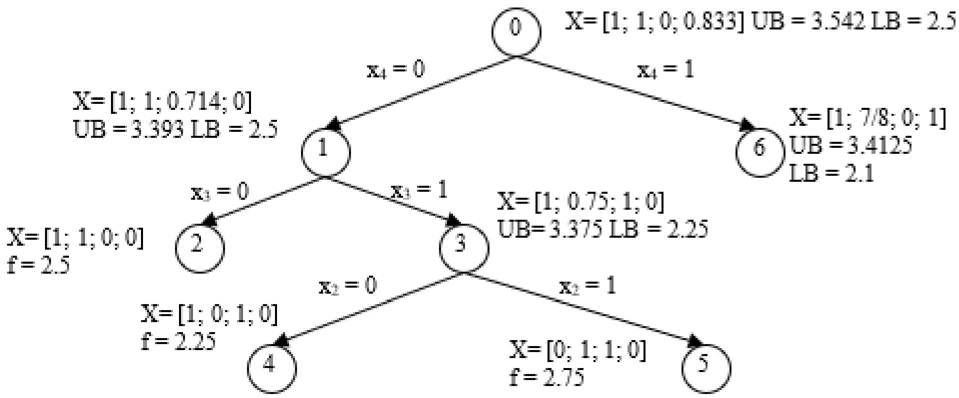


Fig. 5. Relaxing the constraint of the Node 6

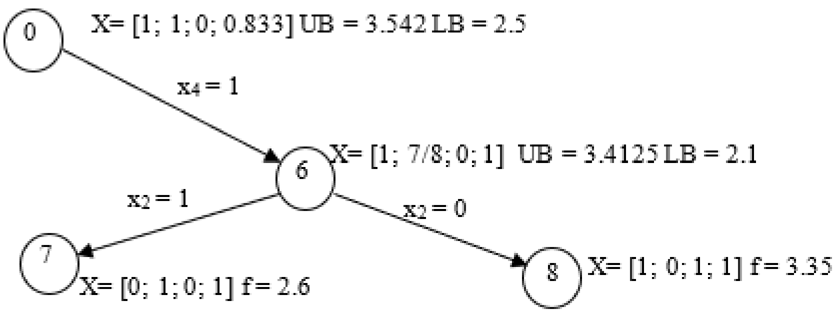


Fig. 6. Investigation of the branches of the Node 6

$$x_{i,j} = \frac{1+(-1)^{\left\lceil \frac{j}{2^{i}-1} \right\rceil}}{2} \tag{7}$$

In Eq. 7 $\lceil \; \rceil$ symbol is the round up operator. The equation is implemented for each 4 design variable for j = {1,...,16} and each possible decision combination is produced. Eq. 5 is implemented by the "*=SUMPRODUCT($B$2:$E$2,B6:E6)-IF(SUM(B6:E6)>3,50,0)-IF(SUMPRODUCT($B$1:$E$1,B6:E6)>18,50,0)*" command. *SUMPRODUCT* function implements Eq. 1 by summing each p_i*x_i multiplication. The first if statement gives 50 million $ penalty if the number of housing construction is more than 3, and the second if statement computes the summation of the executed construction alternatives and applies 50 million $ penalty if the limitation on the budget is violated. The result is given as screenshot for representation in Fig. 7. Same solution is obtained in negligible computation time with 12 kB file size.

3.2. Case problem 2

A contractor considers executing housing projects in Çöşnük, Fahri Kayhan, Bostanbaşı, Sıtmapınarı and Tecde districts. The costs of these constructions will be 5, 8, 7, 6 and 9 million $ respectively. The expected profits of the mentioned projects are 1, 1.5, 1.25, 1.1 and 1.7 million $ respectively. Tecde district is far from the city center and the construction at this region is preferable if the contractor has a nearby construction activity. The selection of Tecde district is possible if the contractor has a construction at Bostanbaşı. The contractor will not prefer construction at Tecde and Çöşnük together because of the long distance between the two regions. If the contractor's budget is 25 million $ and can execute at most three constructions what is the highest profit that the contractor can achieve?

Variables of the problem are represented as; x_1 Construct in Çöşnük or not (1 or 0), x_2 Construct in Fahri Kayhan or not (1 or 0), x_3 Construct in Bostanbaşı or not (1 or 0), x_4 Construct in Sıtmapınarı or not (1 or 0), x_5 Construct in Tecde or not (1 or 0). The problem is expressed as below.

$$\max F(X) = 1x_1 + 1.5x_2 + 1.25x_3 + 1.1x_4 + 1.7x_5$$

(objective function)

$$5x_1 + 8x_2 + 7x_3 + 6x_4 + 9x_5 \leq 25$$

(restriction on the budget)

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 3$$

(restriction on the number of constr.)

$$x_i \in \{0, 1\} \quad j = \{1,\ldots,5\} \quad \text{(decision variables are 0-1)}$$

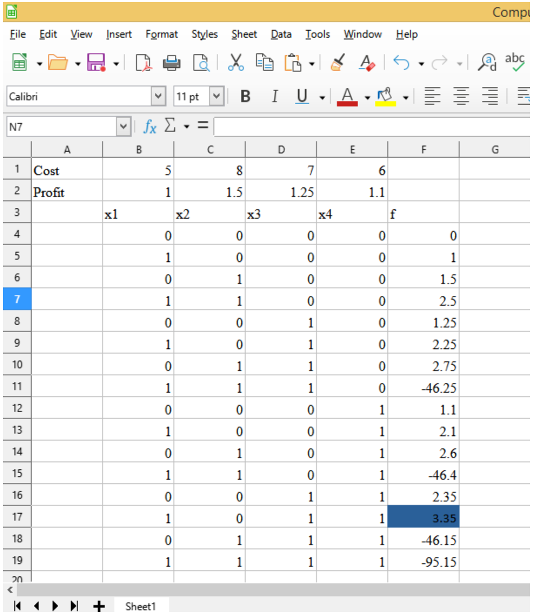
$$x_3 + x_5 \leq 0 \qquad \text{(restriction of Bostanbaşı-Tecde)}$$

$$x_1 + x_5 \leq 1 \qquad \text{(restriction of Çöşnük-Tecde)}$$

The variables and the restriction equations are normalized as following.

$$\min f(X) = -1x_1 - 1.5x_2 - 1.25x_3 - 1.1x_4 - 1.7x_5$$

$$\min f(X) = -1x_1 - 1.5x_2 - 1.25x_3 - 1.1x_4 - 1.7x_5 - f$$



	A	B	C	D	E	F	G
1	Cost	5	8	7	6		
2	Profit	1	1.5	1.25	1.1		
3		x1	x2	x3	x4	f	
4		0	0	0	0	0	
5		1	0	0	0	1	
6		0	1	0	0	1.5	
7		1	1	0	0	2.5	
8		0	0	1	0	1.25	
9		1	0	1	0	2.25	
10		0	1	1	0	2.75	
11		1	1	1	0	-46.25	
12		0	0	0	1	1.1	
13		1	0	0	1	2.1	
14		0	1	0	1	2.6	
15		1	1	0	1	-46.4	
16		0	0	1	1	2.35	
17		1	0	1	1	3.35	
18		0	1	1	1	-46.15	
19		1	1	1	1	-95.15	

Fig. 7. Screenshot of the complete enumeration process

$$5x_1 + 8x_2 + 7x_3 + 6x_4 + 9x_5 + x_6 = 25$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_7 = 3$$

$$-x_3 + x_5 + x_8 = 0$$

$$x_1 + x_5 + x_9 = 1$$

$$x_1 + x_{10} = 1$$

$$x_2 + x_{11} = 1$$

$$x_3 + x_{12} = 1$$

$$x_4 + x_{13} = 1$$

$$x_5 + x_{14} = 1 \qquad x_j \geq 0 \qquad j = \{1,\ldots,14\}$$

3.2.1. Solution of case problem 2 by simplex algorithm

The simplex table illustrated in Table 7 is prepared according to the restrictions of the problem. The variable x_5 is dependent on the x_3 therefore it is skipped and the solution starts with executing the variable x_2.

The next high profit providing construction alternative is x_3, and it is also entered. The obtained situation is shown is Table 8.

In this case the variable x_8 is removed and the variable x_5 is entered. The budget is available and the solution provides 4.45 million $ profit. The obtained solution cannot be improved and the iteration stops.

Table 7. The initial situation of the second case study problem

Basic Variables	Variables																$\frac{b_i}{a_{ij}}$
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	-f	b_i"	
x_6	5	8	7	6	9	1	0	0	0	0	0	0	0	0	0	25	25/8
x_7	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	3	3/1
x_8	0	0	-1	0	1	0	0	1	0	0	0	0	0	0	0	0	X
x_9	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	X
x_{10}	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	X
x_{11}	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1/1
x_{12}	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	X
x_{13}	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	X
x_{14}	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	X
-f	-1	-1.5	-1.25	-1.1	-1.7	0	0	0	0	0	0	0	0	0	1	0	

Table 8. Execution of x_2 and x_3 of the second case study problem

Basic Variables	Variables																$\frac{b_i}{a_{ij}}$
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	-f	b_i"	
x_6	5	0	0	6	9	1	0	0	0	0	-8	-7	0	0	0	10	10/9
x_7	1	0	0	1	1	0	1	0	0	0	-1	-1	0	0	0	1	1/1
x_8	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	1/1
x_9	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	1/1
x_{10}	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	X
x_2	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	X
x_3	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	X
x_{13}	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	X
x_{14}	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	1/1
-f	-1	0	0	-1.1	-1.7	0	0	0	0	0	1.5	1.25	0	0	1	2.75	

3.2.2. Solution of case problem 2 by branch-and-bound algorithm

The variables are sorted according to their profit to cost ratios as x_1, x_5, x_2, x_4, and x_3. Execution of x_1 prevents x_5 and x_3 for this reason x_1 is not executed in the initial solution. The solution starts with executing x_5, x_2 and x_3 leaving 1 million \$ available budget. The restriction on the x_4 is relaxed at the Node 0 and X = [0, 1, 1, 1/6, 1] initial solution is obtained. The relaxation makes the upper bound 4.633. In Fig. 8 the relaxation process is investigated at the Nodes 1 and 2.

In Node 1 x_4 is not executed. This solution does not violate any restriction and the node is not investigated further. The obtained solution provides 4.45 million \$ local maximum. In Node 2 execution of x_4 is investigated by relaxing x_2. The relaxation provides the solution X=[0, 3/8, 1, 1, 1] with 4.6125 million \$ profit which is higher than the current best solution. The violation of the restriction on x_2 is investigated at Nodes 3 and 4. In Node 3, $x_2 = 0$ case which provides 4.05 million \$ profit is investigated. Obtained solution does not violate any restriction and the node 3 is not investigated further.

At Node 4, x_2 and x_4 have to be executed. The remaining budget is used for x_5 and the restriction on the x_3 is relaxed. The relaxation provides 4.6571 million \$ profit which is higher than the current best solution. In node 5, $x_3 = 0$ case is examined which prevents the execution of x_5 as well.

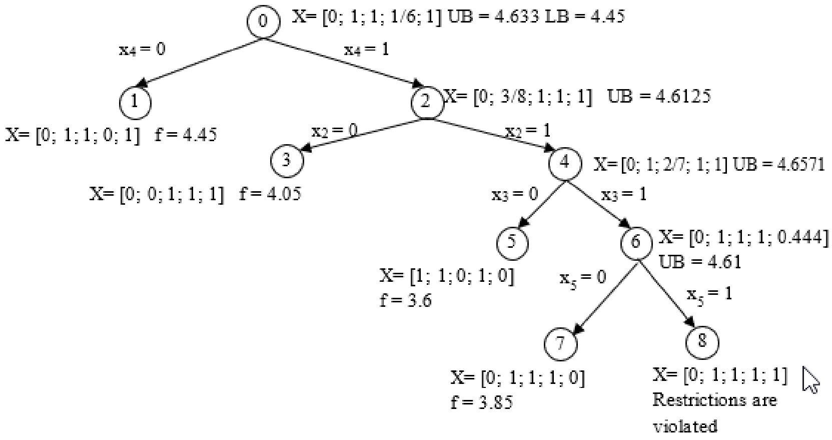


Fig. 8. Branch and bound schema of the second case study problem

The solution provided 3.6 million \$ profit which is worse than the current best and the node is not investigated further since there is not any violated restriction. In node 6 the execution of x_1 should be examined since the objective function of the relaxed case is higher than the current best. Node 7 investigates the case in which x_1 is not executed. This situation provides 3.85 million \$ profit which is worse than the current best. Node 8 investigates the case in which x_1 is executed. However, this case violates the restrictions and the solution is not feasible. There is not any feasible solution alternatives and the obtained X= [0; 1; 1; 0; 1] solution with 4.45 million \$ profit.

3.2.3. Solution of case problem 2 by complete enumeration

Complete enumeration examines 32 combinations by looping i = {1,…,5} and j = {1,…,32}. Global optimum solution is obtained within negligible computation time with 14kB file size. The evaluation function is computed by writing the given expression to the formula bar; "*=SUMPRODUCT($B$2:$F$2,B5:F5)-IF(SUM(B5:F5)>3,50,0)-IF(SUMPRODUCT($B$1:$F$1,B5:F5)>25,50,0)-IF(-D5+F5>0,50,0)-IF(B5+F5>1,50,0)*". The last two if statements are added when compared with the previous problem. The first if statement gives penalty if x_5 is selected without executing x_3. The last if statement gives penalty if x_1 and x_5 are executed together. Complete enumeration provides the same optimum solution which is provided by the simple and the branch-and-bound algorithms.

3.3. Case problem 3

A contractor considers executing housing projects in Yakınkent, Bostanbaşı, Fahri Kayahan, Çöşnük, Temelli, Station Junction, Old Malatya, Yeşilyurt, Paşaköşkü, Taştepe, Tecde, Orduzu, Yakınca and Kernek districts. The costs of these constructions will be 8.5, 10, 12, 8, 8.5, 11, 7.5, 10.5, 5, 4, 13, 6.5, 7 and 6 million \$ respectively. The expected profits of the mentioned projects are 1.75, 2, 2.5, 1.5, 1.75, 2.1, 1.4, 2, 1, 1.25, 2.25, 0.9, 1 and 1.25 million \$ respectively. The selection of Tecde district is possible if the contractor has a construction at Bostanbaşı. Yakınca and Çöşnük are mutually exclusive alternatives. If the contractor's budget is 50 million \$ and can execute at most six housing constructions what is the highest profit that the contractor can achieve?

Variables of the problem are represented as; x_1 Construct in Yakınkent, x_2 Construct in Bostanbaşı, x_3 Construct in Fahri Kayahan, x_4 Construct in Çöşnük, x_5 Construct in Temelli, x_6 Construct in Station Junction, x_7 Construct in Old Malatya, x_8 Construct in Yeşilyurt, x_9 Construct in Paşaköşkü, x_{10} Construct in Taştepe, x_{11} Construct in Tecde, x_{12} Construct in Orduzu, x_{13} Construct in Yakınca and x_{14} Construct in Kernek. The problem is expressed as below.

$$\max f(X) = 1.75x_1 + 2x_2 + 2.5x_3 + 1.5x_4 + 1.75x_5 + 2.1x_6 + 1.4x_7 + 2x_8 + 1x_9 + 1.25x_{10} + 2.25x_{11} + 0.9x_{12} + 1x_{13} + 1.25x_{14}$$

subjected to:

$$8.5x_1 + 10x_2 + 12x_3 + 8x_4 + 8.5x_5 + 11x_6 + 7.5x_7 + 10.5x_8 + 5x_9 + 4x_{10} + 13x_{11} + 6.5x_{12} + 7x_{13} + 6x_{14} \leq 50$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} \leq 6$$

$x_j \in \{0, 1\}$ $\quad$ $j = \{1,\ldots,14\}$

$-x_2 + x_{12} \leq 0$ $\quad$ (restriction of Bostanbaşı-Tecde)

$x_4 + x_{13} \leq 1$ $\quad$ (restriction of Çöşnük-Tecde)

The variables and the restriction equations are normalized as following.

$$\min f(X) = -1.75x_1 - 2x_2 - 2.5x_3 - 1.5x_4 - 1.75x_5 - 2.1x_6 - 1.4x_7 - 2x_8 - 1x_9 - 1.25x_{10} - 2.25x_{11} - 0.9x_{12} - 1x_{13} - 1.25x_{14} - f$$

subjected to:

$$8.5x_1 + 10x_2 + 12x_3 + 8x_4 + 8.5x_5 + 11x_6 + 7.5x_7 + 10.5x_8 + 5x_9 + 4x_{10} + 13x_{11} + 6.5x_{12} + 7x_{13} + 6x_{14} + x_{15} = 50$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{16} = 6$$

$-x_2 + x_{12} + x_{17} = 0$

$x_4 + x_{13} + x_{18} = 1$

$x_k + x_{k+18} = 1$ $\quad$ $k = \{1,\ldots,14\}$

$x_j \geq 0$ $\quad$ $j = \{1,\ldots,32\}$

The profit to cost ratios of the construction alternatives are given in Table 9.

3.3.1. Solution of case problem 3 by simplex algorithm

The simplex table contains 32 columns therefore; the simplex tables cannot be presented. The problem starts with zero construction and zero profit case. The variables are included to the problems according to their profit per cost ratio. Variable x_{10} has the highest ratio and it is included first. The steps of the simplex problem are given in Table 10.

Table 9. Maximum profit per cost ratios of the alternatives

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
Cost	8.5	10	12	8	8.5	11	7.5	10.5	5	4	13	6.5	7	6
Profit	1.75	2	2.5	1.5	1.75	2.1	1.4	2	1	1.25	2.25	0.9	1	1.25
P/C	0.2059	0.2000	0.2083	0.1875	0.2059	0.1909	0.1867	0.1905	0.2000	0.3125	0.1731	0.1385	0.1429	0.2083

Table 10. Solution path of the simplex method.

Step	Change	Profit	Cost	Total Const.
initial	N.A.	0	0	0
1	Include x_{10}	1.25	4	1
2	Include x_3	3.75	16	2
3	Include x_{14}	5	22	3
4	Include x_1	6.75	30.5	4
5	Include x_5	8.5	39	5
6	Include x_2	10.5	49	6
7	Include x_9 Exclude x_2	9.5	44	6
8	Include x_6 Exclude x_9	10.6	50	6

The first 5 steps include the variable with the highest profit to cost ratio among the present variables which are not included to the design parameter set. In the sixth step the variable with the highest profit/cost ratio, x_2, is included. This makes the number of total construction 6 and no additional construction opportunity exist. The remaining budget does not allow construction of an additional facility. The obtained solution might be improved since the remaining budget can be used for a more expensive alternative with a lower profit/cost ratio. Therefore x_2 is included and x_9 is included which reduces the total profit to 9.5 million \$. The available 6 million \$ budget and the restriction on the number of construction does not allow construction of an additional alternative. Then x_9 is excluded and the alternative with the highest profit to cost ratio among the remaining alternatives, x_6 is included. The obtained design variable set, uses up all of the budget and the number of construction alternatives. Therefore, the obtained solution is optimum and cannot be improved.

3.3.2. Solution of case problem 3 by branch and bound algorithm

The initial solution with branch and bound algorithm includes the variables with the highest profit to cost ratio in descending order. The initial variable set includes x_1, x_2, x_3, x_5, x_{10}, x_{14} and $0.2x_9$ when the restrictions are relaxed. The obtained solution provides 10.7 million \$ and 10.5 million \$ profit values as upper bound and lower bound respectively. The lower bound is obtained by rounding down the coefficient of x_9. The situation of the x_9 alternative is examined at nodes 1 and 2. In node 1 x_9 is removed and the alternative with highest profit per cost ratio x_6 is included by relaxation. The mentioned modification provides 10.69 million \$ upper bound and does not change the lower bound. In node 2 x_9 is executed. To prevent violation of the restrictions the alternative with the lowest profit per cost ratio, x_2, is removed from the parameter set. The relaxation provides 10.70 million \$ upper bound and 9.5 million \$ lower bound. Node 2 has higher upper bound value than Node 1so the solution continues with this node.

Node 2 is bounded on x_2. In Node 3 x_2 is executed by relaxing the variable x_1. Variables x_1 and x_5 have the same profit to cost ratio and the selection among x_1 and x_5 is made randomly. Relaxation of x_1 provides 10.68 million \$ upper bound. In Node 4 x_2 is removed and x_6, the variable with the highest profit to cost ratio is relaxed. This provides 10.65 million \$ upper bound. The solution continues from node 1 which has the highest upper bound. The variable x_6 is removed at Node 5 and the variable x_8 is included by relaxation which provides 10.69 million \$ upper bound. Variable x_6 is executed at Node 6 by removing the alternative x_2 which has the lowest cost to benefit ratio. In this case the budget and the number of construction alternative restrictions are active and no restriction is relaxed. The obtained solution is feasible and node 6 is not branched on further. The variable set provides 10.6 million \$ profit.

The branching of node 5 provides the nodes 7 and 8. In node 7 x_8 is removed and variable x_4 is included by relaxation. Upper bound of this node is computed as 10.69 million \$. In node 8 the variable x_8 is included and the variable x_2 is removed. The upper bound of the relaxed case is lower than the current solution which means that branching on this node cannot improve the current best. The node 7 have two branches due to the space limitations the branch with $x_4 = 1$ is not shown. In order to examine this situation x_5 is relaxed which provides 10.53 and 10.25 million \$ upper and lower bounds respectively.

Nodes 10, and 11 are formed by the branches of the node 3. The relaxation of the mentioned nodes does not improve the current best solution and the branch and bound provides 10.6 million \$ profit. The branch-and-bound schema of the problem is shown in Fig. 9.

3.3.3. Solution of case problem 3 by complete enumeration

Complete enumeration examines 16384 combinations by substituting $i = \{1,..,14\}$ and $j = \{1,\ldots,16384\}$ to Eq. 7. Global optimum solution is obtained within less than 5 seconds of computation time with 1390 kB file size.

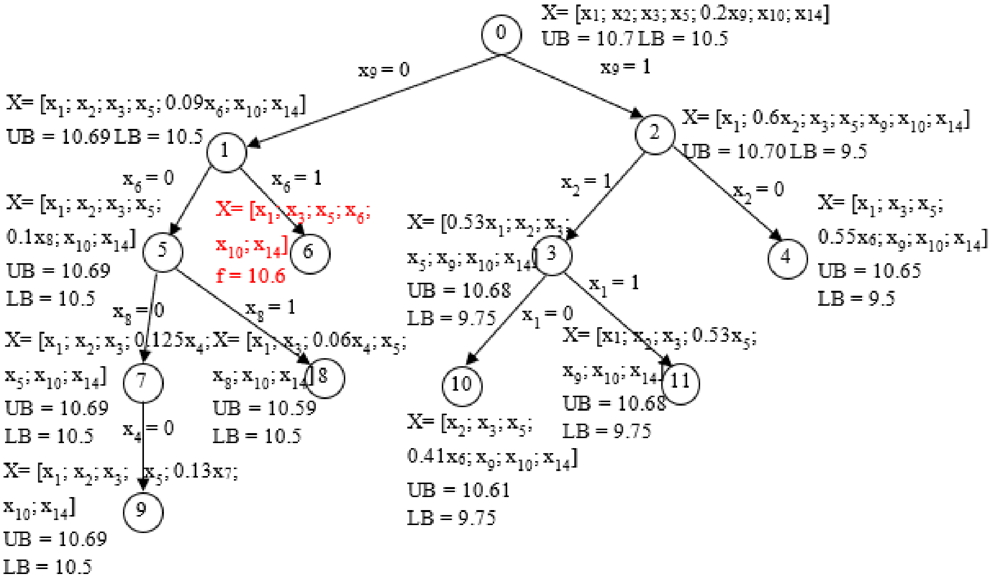


Fig. 9. Branch and bound schema of the third case study problem

The evaluation function is computed by writing the given expression to the formula bar; "*=SUMPRODUCT(B$2:O$2,B6:O6)-IF(SUM(B6:O6)>6,50,0)-IF(SUMPRODUCT(B$1:O$1,B6:O6)>50,50,0)-IF(-C6+M6>0,50,0)-IF(E6+N6>1,50,0)*". The complete enumeration provides the optimum solution which is 10.6 million $. The implementation of complete enumeration is easier than the simplex and the branch-and-bound methods. On the other hand, complete enumeration may end up with too large file sizes and long computation duration if more construction alternatives exist.

4. Discussion of results

In this study housing contractor profit maximization problem is defined and solved by simplex algorithm, branch-and-bound algorithm and complete enumeration. The problem considers the limitations on the budget of the contractor, maximum number of housing project that the contractor can execute and the mutually exclusive cases between the housing projects. The decision variables are binary where they can have 0 or 1 values which make the problem suitable to solve by simplex or branch-and-bound algorithms.

The complexity of the problem depends on the number of decision variables and the number of restrictions. The first case study problem ends up with 10 variables when the problem is written in normal form. The formulation of the problem to solve by simplex or branch-and-bound algorithm requires normalization process which necessitates theoretical knowledge on the optimization algorithms. For this reason, the construction sector may not be eager to define and solve the examined problem. Therefore, the problem is also solved by complete enumeration to avoid the implementation of optimization algorithms. The search domain of the problem is very narrow when the memory and computational capacity of the computers are concerned. Therefore, the implementation of the complete enumeration can be possible for even larger search domains. Complete enumeration is implemented for the solution of small and medium scale resource leveling and resource constraint project scheduling problems [11-13]. However, in the literature optimization problems are also solved by meta-heuristic algorithms [14, 15]. The proposed housing contractor profit maximization problem is not very complex and the utilization of meta-heuristic algorithms is not compulsory.

The largest case study problem contains 14 design variables which makes 16384 different decision alternatives. The mentioned search domain can be enumerated within seconds by an average configured desktop computer. On the other hand, the problem can be more complicated for large-scale housing contractors who make business at international scale. In this case, the number of housing construction opportunities can be significantly more than 14 and the size of the search domain may exceed millions of decision alternatives. In this case, complete enumeration with spreadsheet applications may not be applicable and implementation of simplex, branch-and-bound or meta-heuristic methods might be necessary.

The third case study problem became an indicator for the comparison of the solution methods. The branch-and-bound is the most difficult algorithm to implement. The automation of the branches is not straightforward because of the restrictions among the variables. In addition to this, the number of evaluations is higher than the simplex algorithm. The proposed profit maximization problem can be solved by simplex algorithm if complete enumeration will not be implemented.

The examined problem is 0-1 integer programming problem which has numerous applications in operations research. In this study, a new implementation of 0-1 integer programming is executed by analyzing the housing construction alternatives of a contractor. The analyzed problem can be modified and adopted for the situations of the other contractors such as highway or earthwork contractors. However, the construction opportunities of the aforementioned contractors cannot be as many as the construction opportunities of the housing contractors. Therefore, the best construction alternatives can be determined without implementing any optimization process.

In this study, the housing projects are considered as fixed and any possible design alternatives are not considered. The various design alternatives for a housing project located at a certain project can alter the cost and the profit. The design alternatives can be construction of detached houses, apartment, green certified building and many more. The inclusion of the mentioned effect would enlarge the search domain of the problem.

5. Conclusion

In this study, profit maximization problem for the housing contractors are formulated and solved by simplex, branch-and-bound and complete enumeration. All of the algorithms obtained global optimum of the problem. The implemented algorithms can also provide the global optimum for larger size of the problem. The contractors can define and solve the problem by examining the case study problems and maximize their profit by considering their financial situation and workforce condition. The solution of the defined problem can be beneficial for the housing contractors to maximize their profit. In addition to this, the defined problem can be utilized to illustrate the implementation of simplex and branch-and-bound algorithms at graduate construction management courses.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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