Journal of Construction Engineering, Management & Innovation
2022 5(1):45-63
DOI 10.31462/jcemi.2022.01045063

RESEARCH ARTICLE

Thermo-Economic optimization for saving energy in residential buildings using population-based optimization techniques

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Article History

Received 02 March 2022
Accepted 28 March 2022

Keywords

Thermo-economic optimization,
Population-based methods
Energy saving

Abstract

The total heating cost of residential buildings corresponds to a significant part of the energy consumption of countries. Designing cost-efficient residential buildings in terms of energy gains importance. In the current study, it is targeted to take a beneficial step that will contribute to this issue. In this respect, the decisive components for the calculation of the total heating cost of insulated buildings (i.e. the fuel type, insulation material, and insulation thickness) are determined optimally. For this aim, an optimization model is utilized in which the total heating cost based on life cycle cost analysis is considered as the objective function. The design variables are selected from both continuous and discrete spaces and they are dependent on each other. For solving the problem with such a complex search domain, different well-established non-gradient and population-based optimization techniques are utilized. These methods do not require the information of the objective functions so they can be used widely in solving different optimization problems. In addition, multivariable thermo-economic optimization for minimizing the total heating cost of the insulated building with the selected methods presents the effect and power of the population-based methods in solving different engineering optimization problems. The considered methods are tested on unconstrained mathematical functions and thermo-economic optimization of five distinct locations in the Aegean region of Turkey. The comparative assessments are reported and discussed in detail. Different analyses are employed for evaluating the performance of the optimization techniques. According to the archived outcomes, the utilized optimization techniques present acceptable performance in handling both discrete and continuous design variables.

1. Introduction

The unsustainable energy sources are going to expire. Finding more efficient ways for saving energy and using renewable energies gained more significance in recent years. A considerable portion of the energy is utilized in heating residential buildings. In this respect using the most proper fuel and insulating system for a cost-efficient design is necessary. In residential buildings, a considerable part of heat losses occurs from the exterior walls of the buildings. For reducing these losses, applying an insulating system to the exterior walls is a practical remedy [1, 2]. Determining the thickness of the insulation material is an optimization

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problem with a single variable. There are several studies those are applied gradient-based techniques for solving the mentioned optimization problem [3-11]. In these studies for thermal modeling of the insulated building, the Degree-Day (DD) values of the geographic location are considered. For economic modeling of the system, the Life Cycle Cost Analyze (LCCA) technique is employed. Consequently, the total heating cost of the insulated building is considered the objective function of the optimization problem. In a more recent study, a simultaneous multivariable approach is employed [12]. Besides the insulation thickness, the type of the fuel and the type of the insulation material are considered simultaneously as the design variables of the optimization problem. For solving the optimization problem, a non-gradient and population-based optimization technique are utilized. For saving more energy and cost in buildings, carrying out studies with more effective optimization techniques for solving these kinds of problems acquires more importance.

Optimization techniques are divided into two main groups; gradient-based and non-gradient-based techniques. Non-gradient methods do not require the gradient information of the objective function [13]. Also, they are not sensitive to the initial condition of the searching process and step sizes. They can start from any arbitrary or improper position and without easily trapping to the local optima [14, 15]. There are two search patterns for scanning the search domain during the optimization process; Local search and Global search. In optimization methods, making a balance between two strategies is vital and this affects the search performance of the optimization algorithm [16, 17]. There are different internal parameters and algorithms for providing a more effective search capability in different methods. There are main categories for the classification of non-gradient techniques. These categories are evolutionary-based, physics-based, nature-inspired, and social-based techniques. In the current study, for giving an exhaustive point of view for the readers, different methods are selected from these categories [18-20]. The selected optimization techniques are; Differential Evolution (DE), Ions Motion Optimization (IMO), Integrated Particle Swarm Optimization (iPSO), Harris Hawks Optimization (HHO), and Interactive Search Algorithm (ISA).

The thermo-economic optimization for the insulated building is studied for a single variable (i.e. insulation thickness) and the conventional gradient-based approach is utilized for solving it [3, 5, 11, 21, 22]. Considering the fuel type and the insulation material type gives a realistic dimension to the problem. In addition, utilizing recently developed non-gradient population-based methods as the optimizer tool contributes to the technical literature. For this target, in the current study, minimizing the total heating cost of the insulated building with simultaneous multi-variables is investigated. For minimizing the total heating cost of insulated residential buildings six different non-gradient and population-based optimization methods are utilized. In the case studies, five different locations from the Aegean region of Turkey are selected. Also, for giving a more comprehensive range of view for the researchers six distinct mathematical functions selected from CEC2017 with different properties are tested. The performance of these methods is tested and compared via accuracy and stability, convergence behavior, complexity, and non-parametric statistical analyses.

2. Optimization methods

In the current section, the selected optimization algorithms are briefly described. All selected algorithms are non-gradient and population-based techniques. The selected optimization methods and their descriptions are chronologically listed in Table 1. These methods are not dependent on the start point of the search process and the step sizes are not determinative in these methods. It is not necessary to define a continuous objective function and its gradients. These advantages cause the algorithms to not trap easily in any local optima and the ineffective iterations decrease. Each agent is a potential solution in these kinds of methods.

Table 1. Parameter setting for the applied algorithms

Algorithm	Year	Parameter	Description
DE [23]	1997	$F \in \{0,2\}$	Differentiation amplification factor (F)
IMO [24]	2015	$\varphi_1, \varphi_2 = rand\ (-1, 1)$	Movement factor (φ_1, φ_2) Anion (A) , Cation (C)
iPSO [25]	2017	$\alpha = 0.4, C_3 = C_4 = 1, C_2 = 2$	Acceleration factors (C_1,C_2, C_3,C_4)
HHO [26]	2019	$E_0 = 2rand(0,1) - 1$ $J = 2(1 - rand(0,1))$	The initial state of prey's energy (E_0) Random jump strength (J)
ISA [27]	2019	$\tau = 0.3$	Tendency factor(τ)
GTOA [28]	2020	$F \in \{0,2\}$	Teaching factor (F)

The agents initialize from any arbitrary location in the search domain and their location is improved gradually via evaluating the objective function value in each iteration. These methods are chronologically explained in the following sub-sections.

2.1. Differential evolution (DE)

Differential Evolution (DE) method has two main phases crossover and mutation. In the mutation phase, a mutant vector, based on three randomly considered agents, is generated for each agent. However, in the crossover phase, each agent is crossed over with its generated mutant vector [23]. Based on the given information the mutation and crossover phases are mathematically formulated as below:

Mutation phase:

$$ {}^{t+1}V_i = {}^{t}X_{r1} + F \cdot ({}^{t}X_{r2} - {}^{t}X_{r3}), \quad where, \quad r1 \neq r2 \neq r3 \neq i \tag{1} $$

Crossover phase:

$$ {}^{t+1}X_{ij} = \begin{cases} {}^{t}V_{ij}\,, & if\ r4 \leq CR\ or\ r5 = j \\ {}^{t}X_{ij}\,, & otherwise \end{cases} \tag{2} $$

r1, *r2* and *r3* are different integers randomly selected from [1, PS], in which PS is the population size. However, *r*4 and *r*5 have been randomly selected from [0, 1] intervals. F is the differentiation amplification factor and is considered to be selected from [0, 2] interval as a real scalar. X_{ij} presents the jth component of the *i*th agent. V_{ij} is the jth component of the ith mutant vector. In addition, *CR* is the crossover constant and it shows whether the probability of mutation for the current agent's components is accepted or not. The work scheme of the DE is presented in its pseudo-code in Table 2.

2.2. Ions motion optimization (IMO)

The treats of the ions in nature, the repulsion and attraction forces within the anions and cations became inspiration sources for the Ions Motion Optimization (IMO). The population size of the algorithm at the inception of the search process must be an even number because agents are divided into two groups anions and cations. There are two distinct search patterns such as liquid phase and solid phase in this method. There are not any random coefficients in the liquid phase of the IMO algorithm, which makes this phase thoroughly non-stochastic. In the solid phase of the algorithm, the exploitation behavior of the algorithm is dominant and the agents move toward the best solution during the search process for preventing the algorithm from trapping in local optima [24]. According to the given information, the mathematical description of the current method is formulated as below:

Liquid phase

$$ \begin{aligned} A_{ij} &= A_{ij} + AF_{ij} \times \left(Cbest_j - A_{ij}\right) \\ C_{ij} &= C_{ij} + CF_{ij} \times \left(Abest_j - C_{ij}\right) \end{aligned} \tag{3} $$

Table 2. The pseudo-code for DE

```
Initialize internal parameters and agents;
while (the termination criteria are not met)
    For each agent calculate the mutation vector from Eq. (1)
    if (the mutation is probable)
        the mutant vector is the new agent based on Eq. (2)
    else if
        t+1X_ij = tX_ij
    end
end
```

Solid-phase

$$\textbf{if}\left(\begin{array}{c} CbestFit \geq \frac{CworstFit}{2} \\ and \\ AbestFit \geq \frac{AworstFit}{2} \end{array}\right)$$

$$\begin{cases} A_i = A_i + \Phi_1 \times (Cbest - 1) & \text{if } rand() > 0.5 \\ A_i = A_i + \Phi_1 \times (Cbest) & \text{if } rand() \leq 0.5 \end{cases} \tag{4}$$

$$\begin{cases} C_i = C_i + \Phi_2 \times (Abest - 1) & \text{if } rand() > 0.5 \\ C_i = C_i + \Phi_2 \times (Abest) & \text{if } rand() \leq 0.5 \end{cases}$$

$$Re - initialized\ A_i\ and\ C_i \quad \text{if} \quad rand() < 0.05$$

Abest, *Cbest*, *AbestFit* and *CbestFit* indicate the best anion, best cation, fitness value for the best anion and fitness value for the best cation, respectively. Φ_1, Φ_2 is uniformly selected from the interval [-1, 1]. *rand ()* is randomly selected from [0, 1] interval. AF_{ij} and CF_{ij} are the force coefficients between the ions and are determined as below:

$$AF_{ij} = \frac{1}{1 + e^{-0.1/AD_{ij}}}$$

$$CF_{ij} = \frac{1}{1 + e^{-0.1/CD_{ij}}} \tag{5}$$

in which

$$D_{ij} = |A_{ij} - Cbest_j|\,, \qquad CD_{ij} = |C_{ij} - Abest_j|$$

AD_{ij} and CD_{ij} are the distance between the *i*th agent and best anion/cation, respectively. For giving a more clear description of the IMO, the pseudo-code of the method is given in Table 3.

2.3. Integrated particle swarm optimization (iPSO)

The integrated Particle Swarm Optimization (iPSO) is the enhanced version of the PSO technique via employing a weighted agent to the algorithm for preventing any local optima trapping. There are two different search strategies for scanning the search domain. In one of these strategies, the new agent moves toward three other agents ($\mathbf{X}^G$, $\mathbf{X}^P$ and $\mathbf{X}^W$). In the other strategy, it moves just toward the gravity center of the population ($\mathbf{X}^W$). The algorithm working scheme is mathematically given below [29]:

$$\begin{aligned} & if\ rand_{0i} > \alpha \\ & {}^{t+1}\boldsymbol{v}_i = \omega \times {}^{t}\boldsymbol{v}_i + (\varphi_{1i} + \varphi_{2i} + \varphi_{3i})({}^{t}\boldsymbol{X}_j^P - {}^{t}\boldsymbol{X}_i) + \varphi_{2i}({}^{t}\boldsymbol{X}^G - {}^{t}\boldsymbol{X}_j^P) + \varphi_{3i}({}^{t}\boldsymbol{X}^w - {}^{t}\boldsymbol{X}_j^P) \\ & if\ rand_{0i} \leq \alpha \\ & {}^{t+1}\boldsymbol{v}_i = \varphi_{4i}({}^{t}\boldsymbol{X}^w - {}^{t}\boldsymbol{X}_i) \\ & Updated\ agent: \\ & {}^{t+1}\mathbf{X}_i = {}^{t}\mathbf{X}_i + {}^{t+1}\mathbf{v}_i \end{aligned} \tag{6}$$

in which

$\varphi_{ki} = C_k \times rand_{ki}$ for $k \in \{0,1,2,3,4\}$.

where $rand_{ki}$ is a random number selected from the interval [0, 1]. The acceleration factors are determined as $C_1 = -(\varphi_{2i} + \varphi_{3i})$, $C_2 = 2$, $C_3 = 1$, and $C_4 = 2$. The weighted agent ($\mathbf{X}^W$) is calculated as below:

$$\begin{aligned} & \boldsymbol{X}^w = \sum_{i=1}^{M} \bar{c}_i^w \boldsymbol{X}_i^P \\ & \bar{c}_i^w = \left(\hat{c}_i^w \Big/ \sum_{i=1}^{M} \hat{c}_i^w\right) \\ & c_i^w = \frac{\underbrace{max}_{1 \leq f \leq M}\left(f(\boldsymbol{X}_{kw}^P)\right) - f(\boldsymbol{X}_i^P) + \varepsilon}{\underbrace{max}_{1 \leq kw \leq M}\left(f(\boldsymbol{X}_{kw}^P)\right) - \underbrace{min}_{1 \leq kw \leq M}\left(f(\boldsymbol{X}_{kw}^P)\right) + \varepsilon} \\ & , i = 1,2, \ldots, M \end{aligned} \tag{7}$$

Table 3. The pseudo-code for IMO

```
Initialize internal parameters and agents;
while (the termination criteria are not met)
    Evaluating all agents
    Determining the best and worst Anions and Cations and force coefficients using Eqs. (3-5)
  Update locations of ions based on the liquid phase as given in Eq. (3)
  if (Mean objective function value of worst ions is equal or smaller than the best ions)
      Perform solid-phase motion based on Eq. (4)
  end
end
```

M is the number of agents and *f(.)* presents the objective function value of the current agent. ε is defined as a tiny value (i.e. 1E-06) to prevent any division by zero condition. For more clarification, the pseudo-code of the iPSO is presented in Table 4.

2.4. Harris hawks optimization (HHO)

Harris Hawks Optimization (HHO) is a nature-inspired optimization technique. It mimics the behavior of an intelligent bird so-called Harris'Hawks in hunting escaping prey. There are three phases in this algorithm; (i) Exploration, (ii) Transition from exploration to exploitation and (iii) Exploitation. In HHO, each agent (Harris' Hawks) is a candidate solution. In the exploration phase, the agents change their locations randomly then the average location of agents is achieved. During the location changings, the prey escapes from the agents and their energy decreases. Due to this fact, the transition phase occurs. Then in the exploitation phase of the search process, the agent moves toward the best location based on the attained information from the transition phase [26]. Based on the given information, the algorithm of HHO is mathematically formulated as below:

Exploration phase:

$$X^{t+1} = \begin{cases} X^t_{rand} - r_1\left|X^t_{rand} - 2r_2X^t\right| & q \geq 0.5 \\ \left(X^t_{prey} - X^t_m\right) - r_3\left(LB + r_4(UB - LB)\right) & q < 0.5 \end{cases} \tag{8}$$

$$X^t_m = \frac{1}{N}\sum_{i=1}^{N} X^t_i$$

The transition from exploration to exploitation:

$$E = 2E_0\left(1 - \frac{t}{T}\right) \tag{9}$$

Exploitation phase:

when $r \geq 0.5$ and $|E| \geq 0.5$

$$X^{t+1} = \Delta X^t - E\left|JX^t_{prey} - X^t\right|$$

$$\Delta X^t = X^t_{prey} - X^t$$

$$J = 2(1 - r_5)$$

when $r \geq 0.5$ and $|E| < 0.5$

$$X^{t+1} = X^t_{prey} - E\left|\Delta X^t\right|$$

when $r < 0.5$ and $|E| \geq 0.5$

$$Y = X^t_{prey} - E\left|JX^t_{prey} - X^t\right|$$

$$Z = Y + S \times LF(D)$$

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}}, \tag{10}$$

$$\sigma = \left(\frac{\tau(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\tau\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}}\right)^{\frac{1}{\beta}}$$

$$X^{t+1} = \begin{cases} Y & if \quad f(Y) < F(X^t) \\ Z & if \quad f(Z) < F(X^t) \end{cases}$$

when $r < 0.5$ and $|E| < 0.5$

$$X^{t+1} = \begin{cases} Y & if \quad f(Y) < F(X^t) \\ Z & if \quad f(Z) < F(X^t) \end{cases}$$

$$Y = X^t_{prey} - E\left|JX^t_{prey} - X^t_m\right|$$

$$Z = Y + S \times LF(D)$$

where r_1, r_2, r_3, r_4, r_5, and q are randomly selected from (0, 1) interval. N is the population size. E and E_0 designate the escaping energy and the initial

Table 4. The pseudo-code for iPSO

```
Initialize agents
while (the termination criteria are not met)
  Calculate the weighted agent X^w using Eq. (7)
    for (each agent)
        if (rand_0i ≤ α)
           Calculate the velocity vector applying Eq.(6)
        else if (rand_0i > α)
           Calculate the velocity vector applying Eq.(6)
        end
      Update the current agent
    end
   confirm weighted agent condition
    if (f(X^w)) < (f(X^G))
       Set X^G = X^w,   (whenever X^w has the less objective function value replaced with X^G)
    end
end
```

the energy of the prey, respectively. T and t are the maximum number of iterations and the current iteration, respectively. r indicates the chance of prey in successfully escaping. LF is the levy flight concept and is utilized to model the real zigzag motions of prey during the escaping. For more clarity, the pseudo-code of HHO is given in Table 5.

2.5. Interactive search algorithm (ISA)

Interactive Search Algorithm (ISA) is another population-based optimization technique. There are two different paradigms for scanning the search domain of the problem, named tracking and interacting search strategies. In the tracking and interacting paradigms, the exploration and exploitation search behaviors of the algorithm are respectively dominant. There is a tendency factor (τ) for regulating the algorithm between these two paradigms. This factor (τ) is determined as 0.3 based on a series of sensitivity analyses [27]. Based on the given information the ISA is mathematically presented as below:

$\underline{\text{if } \tau \geq 0.3 \text{ (Tracking paradigm) :}}$

$$ {}^{t+1}_{Tr}V_i = \omega \cdot {}^{t}V_i + \varphi_1 \odot \left({}^{t}X^P_j - {}^{t}X_i\right) + \varphi_2 \odot \left({}^{t}X^G - {}^{t}X^P_j\right) + \varphi_3 \odot \left({}^{t}X^W - {}^{t}X^P_j\right) \tag{11} $$

$\underline{\text{if } \tau < 0.3 \text{ (Interacting paradigm) :}}$

$$ \begin{aligned} {}^{t+1}_{In}V_i &= \varphi_4 \odot \left({}^{t}X_i - {}^{t}X_j\right), \quad if\ f(X_i) < f(X_j) \\ {}^{t+1}_{In}V_i &= \varphi_4 \odot \left({}^{t}X_j - {}^{t}X_i\right), \quad if\ f(X_i) \geq f(X_j) \end{aligned} \tag{12} $$

Updated agent:

$$ {}^{t+1}X_i = {}^{t}X_i + {}^{t+1}V_i \tag{13} $$

$\varphi_k, k = 1 \ldots 4$ is the acceleration factors and for ith agent are selected uniformly from the interval [0, 1]. $\mathbf{X}_j$, $\mathbf{X}^G$, $\mathbf{X}^P$ and $\mathbf{X}^W$ indicate an arbitrary agent, the best agent, the best agent of the memory and the weighted agent of the population, respectively. ω is the inertia weight and it is considered as 0.4 [27]. $\odot$ is the sign of the Hadamard product. The weighted agent is declared in Eq. (7) before. The pseudo-code of the ISA is presented for more illustration in Table 6.

2.6. Group teaching optimization algorithm (GTOA)

Group Teaching Optimization algorithm (GTOA) is inspired by group teaching approach. In the GTOA, the whole class is considered to be in the normal distribution. There are four phases such as teacher allocation, ability grouping, teacher and student phases. There are just two control parameters as population size and stop criteria in the algorithm [28]. These four phases are mathematically presented as below:

Table 5. The pseudo-code for HHO

```
Initialize agents and internal parameters
while (the termination criteria are not met)
  Calculate the fitness value of agents
  Set X^t_prey as the best location
    for (each agent)
       Update E0 and J and E from Eq (9)
       Utilize r and E values to decide the phase of the algorithm
       Update the location vector using Eq. (10)
    end
end
```

Table 6. The pseudo-code for ISA

```
Initialize agents
while (the termination criteria are not met)
  Calculate the weighted agent X^w using Eq. (7)
    for (each agent)
       if (τ < 0.3)
         Calculate the velocity vector applying Eq.(12)
       else if (τ ≥ 0.3)
         Calculate the velocity vector applying Eq.(11)
       end
      Update the current agent
    end
end
```

Teacher allocation phase:

$$T^t = \begin{cases} x^t_{first} & f\left(x^t_{first}\right) \leq f\left(\dfrac{x^t_{first} + x^t_{second} + x^t_{third}}{3}\right) \\ \dfrac{x^t_{first} + x^t_{second} + x^t_{third}}{3} & f\left(x^t_{first}\right) > f\left(\dfrac{x^t_{first} + x^t_{second} + x^t_{third}}{3}\right) \end{cases} \tag{13}$$

Ability grouping phase:

$$f(x) = \frac{1}{\sqrt{2\pi}\delta} e^{\frac{-(x-u)^2}{2\delta^2}} \tag{14}$$

Teacher phase I:

$$x^{t+1}_{teacher,i} = x^t_i + a \times \left(T^t - F \times (b \times M^t + c \times x^t_i)\right)$$

$$M^t = \frac{1}{N}\sum_{i=1}^{N} x^t_i \tag{15}$$

$$b + c = 1$$

Teacher phase II:

$$x_{teacher,i}^{t+1} = x_i^t + 2 \times d \times (T^t - x_i^t)$$

$$x_{teacher,i}^{t+1} = \begin{cases} x_{teacher,i}^{t+1} & f\left(x_{teacher,i}^{t+1}\right) < f(x_i^t) \\ x_i^t & f\left(x_{teacher,i}^{t+1}\right) \geq f(x_i^t) \end{cases} \quad (15)$$

Student phase:

$$x_{student,i}^{t+1} = \begin{cases} x_{teacher,i}^{t+1} + e \times \left(x_{teacher,i}^{t+1} - x_{teacher,j}^{t+1}\right) + g \times \left(x_{teacher,i}^{t+1} - x_i^t\right) & f\left(x_{teacher,i}^{t+1}\right) < f\left(x_{teacher,j}^{t+1}\right) \\ x_{teacher,i}^{t+1} - e \times \left(x_{teacher,i}^{t+1} - x_{teacher,j}^{t+1}\right) + g \times \left(x_{teacher,i}^{t+1} - x_i^t\right) & f\left(x_{teacher,i}^{t+1}\right) \geq f\left(x_{teacher,j}^{t+1}\right) \end{cases} \quad (16)$$

$$x_i^{t+1} = \begin{cases} x_{teacher,i}^{t+1} & f\left(x_{teacher,i}^{t+1}\right) < f\left(x_{student,i}^{t+1}\right) \\ x_{student,i}^{t+1} & f\left(x_{teacher,i}^{t+1}\right) \geq f\left(x_{student,i}^{t+1}\right) \end{cases}$$

In the teacher allocation phase of the algorithm, x_{first}^t, x_{second}^t, x_{third}^t are the first, second and third best students, respectively. In the ability grouping phase, the normal distribution of the class is presented. In which δ is the standard deviation, x is the value of the normal distribution function and u indicates the average knowledge of the whole class. In the teacher phase of the algorithm, a student learns from the teacher. In this phase, N is the number of students. t designates the current iteration and x_i^t is the ith student at the time t. T^t and M^t are the teacher and the mean of the students at the time t, respectively. a, b, c and d are randomly selected from [0, 1] intervals. F is the teaching factor which can be either 1 or 2 [30]. In the student phase, also, the e and g are randomly selected from [0, 1] intervals. For more clarity, the steps of the GTOA are given in pseudo-code form in Table 7.

3. Thermo-Economic optimization model of the exterior wall of the building

The thermal part of the thermo-economic model of the exterior wall of a residential building is carried out using a thermal resistance approach. In this respect, the wall is considered as a thermal element. As presented in Fig. 1 the layers of the wall are assumed as resistances. For a unit wall surface the heat losses via conduction are calculated as below:

Table 7. The pseudo-code for GTOA

```
Initialize agents
while (the termination criteria are not met)
    determining the first three best and calculating T^t from Eq. (13)
    dividing the population into two groups according to Eq. (14)
        for (each group)
            implementing the teacher phase for the groups by Eq. (15)
            implementing the student phase for the groups by Eq. (16)
        end
        if f(x_teacher,i^(t+1)) < f(x_student,i^(t+1))
            x_i^(t+1) = x_teacher,i^(t+1)
        else if
            x_i^(t+1) = x_student,i^(t+1)
        end
end
```

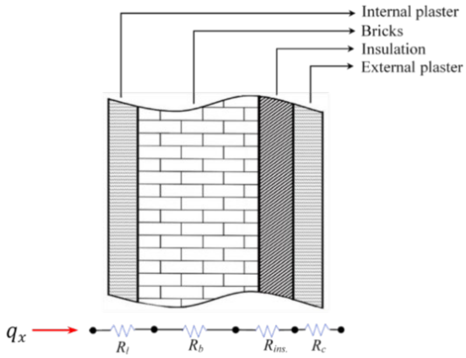


Fig. 1. External wall structure

$$q_x^{"} = -k\frac{dT}{dx} \tag{17}$$

where k is the coefficient of thermal conductivity of the corresponding material.

For each layer of the wall thermal resistance is calculated as below [31]:

$$R_{layer} = \frac{x_{layer}}{k_{layer}} \tag{18}$$

in which x designates the layer thickness. The overall heat transfer coefficient for a multilayer wall is attained as below:

$$U = \frac{1}{R_l + R_b + R_c + R_{ins}} \tag{19}$$

The heating degree-day value is utilized for calculating the annual heating load and energy demand of a building per unit area of the exterior wall as the following [8]:

$$q_{a,h} = 24 \times 3600 \times HDD \times U \tag{20}$$

$$E_{a,h} = \frac{86400 \times HDD \times U}{\eta} \tag{21}$$

where η shows the efficiency of the fuel type utilized for heating. The Life Cycle Cost Analysis (LCCA) technique is applied for developing the economic part of the thermo-economic model. For the unit surface area of the wall the annual heating load is calculated as below:

$$C_{a,h} = \frac{E_{a,h}C_{fuel}}{Hu} \tag{22}$$

where Hu is the heat value of the utilized fuel. The required Present Worth Factor (PWF) in the LCCA technique is calculated as below:

$$\begin{cases} PWF = \dfrac{(1+r)^N - 1}{(1+r)^N \times r} & \begin{cases} r = \dfrac{i-g}{1+g} & i > g \\ r = \dfrac{g-i}{1+i} & i < g \end{cases} \\ PWF = \dfrac{N}{1+i} & i = g \end{cases} \tag{23}$$

where, g and i designate the inflation and interest rates, respectively. N shows the lifetime for the economic assessment. In the current study g=7.91 %, i=8.25 %, and N=10 years [32].

The insulation cost of the wall is the investment cost and is calculated as below:

$$C_{ins.} = x\,C_m \tag{24}$$

Consequently, according to the given information the total heating cost of an insulated building based on the LCCA approach is mathematically presented as below:

$$C_{t,h} = PWF\;C_{a,h} + C_{ins.} \tag{25}$$

It is targeted to minimize the total heating cost of insulated buildings using different optimization algorithms. The objective function of the considered optimization problem is mathematically declared as below:

$$f(\mathbf{X}) = minimize\left(C_{t,h}\right) \tag{26}$$

where the **X** is the vector of the design variables including continuous and discrete components. In the current problem, the design variables are considered as the fuel type, insulation material and insulation thickness.

4. Numerical tests and discussion

In the current section, for evaluating the performance of the selected methods six unconstrained mathematical functions and thermo-economic optimization problems for five cases are solved.

4.1. Unconstrained mathematical functions

For investigating the performance of the selected methods more comprehensively, six different benchmark mathematical functions are tested for the optimization problem. The functions are selected from CEC 2017 in different categories for challenging the methods from different aspects [33]. The properties of the functions, their formulations and 2D plots are given in Table 8. All selected functions are initiated in $[-100, 100]^D$, where D is the problem's dimension which is considered as 30 for all functions.

Different analyses such as accuracy and stability, convergence behavior, complexity, and no-parametric statistical analysis are performed for comparing the performance of the optimization techniques.

Table 8. Properties of the selected benchmark functions

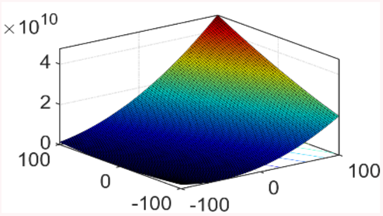
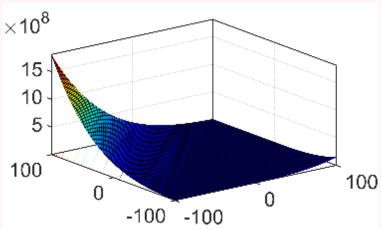
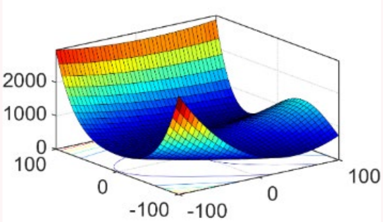
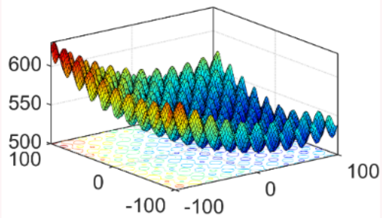
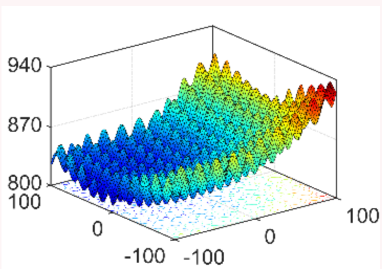
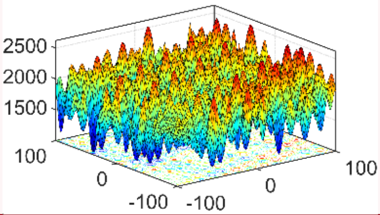
No.	Function name	2D Plot	Formulation**	f^*_{min}
F1	UM, NS, SNR		$L(\boldsymbol{x}) = x_1^2 + 10^6 \sum_{i=2}^{D} x_i^2$ $f(\boldsymbol{x}) = L(\mathrm{M}(\mathrm{x}-\mathrm{o})) + 100$	100
F2	MM, NS		$L(\mathbf{X}) = \sum_{i=1}^{D} x_i^2 + \left(\sum_{i=1}^{D} 0.5x_i\right)^2 + \left(\sum_{i=1}^{D} 0.5x_i\right)^4$ $f(\mathbf{X}) = L(\mathrm{M}(\mathrm{x}-\mathrm{o})) + 300$	300
F3	MM, NS, LOH		$L(\mathbf{X}) = \sum_{i=1}^{D-1} \left(100\left(x_i^2 - x_{i+1}\right)^2 + (x_i - 1)^2\right)$ $f(\mathbf{X}) = L\left(\mathrm{M}\left(\frac{2.048(\mathrm{x}-\mathrm{o})}{100}\right) + 1\right) + 400$	400
F4	UM, NS, SNR		$L(\mathbf{X}) = \sum_{i=1}^{D} \left(x_i^2 - 10\cos(2\pi x_i) + 10\right)$ $f(\mathbf{X}) = L(\mathrm{M}(\mathrm{x}-\mathrm{o})) + 500$	500
F5	MM, NS		$L(\boldsymbol{x}) = \sum_{i=1}^{D} \left(z_i^2 - 10\cos(2\pi z_i) + 10\right) - 20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + 20 + e$ $f(\boldsymbol{x}) = L\left(\frac{5.12(\mathrm{x}-\mathrm{o})}{100}\right) + 800$	800

Table 8. Cont'd

No.	Function name	2D Plot	Formulation**	f^*_{min}
F6	MM, NS, LOH		$L(\boldsymbol{x}) = 418.9829 \times D - \sum_{i=1}^{D} g(z_i)$ $f(\boldsymbol{x}) = L\left(\frac{1000(\boldsymbol{x}-\boldsymbol{o})}{100}\right) + 1000$	1000

*: UM: Uni-Modal, MM: Multi-Modal, NS: Non-Separable, LOH: Local optima's number is huge, A: Asymmetrical, SP: Separable, SNR: Smooth but narrow ridge

**: Operators of o, z, and M are given in detail in the CEC2017 database in: https://github.com/P-N-Suganthan/CEC2017-BoundContrained [33]

4.1.1. Accuracy and stability analysis

The accuracy and stability solutions for the selected functions attained using different optimization methods are reported comparatively in the current section. The achieved results for this analysis are presented in Table 9. Based on the reported optimal outcomes, ISA obtained promising results in both accuracy and stability aspects for all cases. iPSO comes in the second position after ISA. The reason for this observation is that the ISA utilizes both local and global search behaviors during the optimization process and this prevents the algorithm from trapping in any local optima.

4.1.2. Convergence behavior analysis

For observing the search performance of the selected methods more accurately, the convergence rate diagrams for the selected test functions are plotted in Fig. 2. According to the illustrated diagrams, the ISA technique presented the fastest convergence rate than the other methods in achieving the optimal result. This fact designates that the ISA utilizing the interacting and tracking search behaviors simultaneously during the optimization process reduces the ineffective iterations, expedites the convergence rate of the algorithm, and improves the performance of the algorithm.

4.1.3. Complexity analysis

In the current section, the complexity of the selected optimization methods is tested and reported. For this aim, four periodic terms (i.e. T_0, T_1, T_2 and $\hat{T}_2$) are calculated [34]. T_0 is the required time for running following code:

```
for i=1:1000000
      x= 5.55 (x is double);
      x=x + x;
       x=x./2;
      x=x*x;
      x=sqrt(x);
      x=ln(x);
      x=exp(x);
      y=x/x;
end
```

(27)

T_1 is the required time for 200000 evaluation of the desired function [34]; T_2 is the total time required for optimizing the selected function with distinct dimensions (which are taken as D=30 and D=50); $\hat{T}_2$ is defined as the average of all obtained $T2$ values for the current function and dimension. The F6 because of its complexity is selected for the current analysis. The acquired results are presented in Table 10 and Table 11. According to the given outcomes for dimension 30, DE comes before ISA. This fact is due to ISA's search behavior balancing process; although it is less time-consuming than other selected methods. Considering the performance of the ISA from accuracy, stability and convergence rate aspects, this little difference will be negligible. For dimension 50, ISA exceeds other methods.

Table 9. Optimal results for selected test functions

Function	Value	DE	IMO	iPSO	HHO	ISA	GTOA
F1	Mean	4.12E+03	8.91E+04	3.91E+03	6.14E+06	5.19E+02	7.41E+04
	Std.	2.25E+02	7.20E+03	5.01E+01	7.68E+04	3.45E+01	8.21E+03
F2	Mean	8.33E+04	5.13E+03	4.55E+03	5.89E+03	3.76E+03	4.99E+03
	Std.	2.91E+04	6.00E+02	7.19E+01	4.51E+02	6.21E+01	5.88E+02
F3	Mean	4.96E+02	4.90E+02	4.65E+02	5.01E+02	4.51E+02	4.79E+02
	Std.	4.48E+01	5.39E+01	2.01E+01	1.22E+02	5.25E+00	2.71E+01
F4	Mean	6.66E+02	6.15E+02	5.71E+02	6.00E+02	5.51E+02	5.80E+02
	Std.	6.51E+01	5.01E+01	2.11E+01	4.41E+01	1.81E+01	3.37E+01
F5	Mean	9.77E+02	9.17E+02	8.41E+02	8.68E+02	8.39E+02	9.01E+02
	Std.	4.74E+01	3.91E+01	2.69E+01	4.51E+01	2.03E+01	3.66E+01
F6	Mean	5.45E+03	5.47E+03	4.99E+03	5.35E+03	4.12E+03	4.28E+03
	Std.	4.78E+02	5.11E+02	4.16E+02	7.45E+02	2.98E+02	4.86E+02

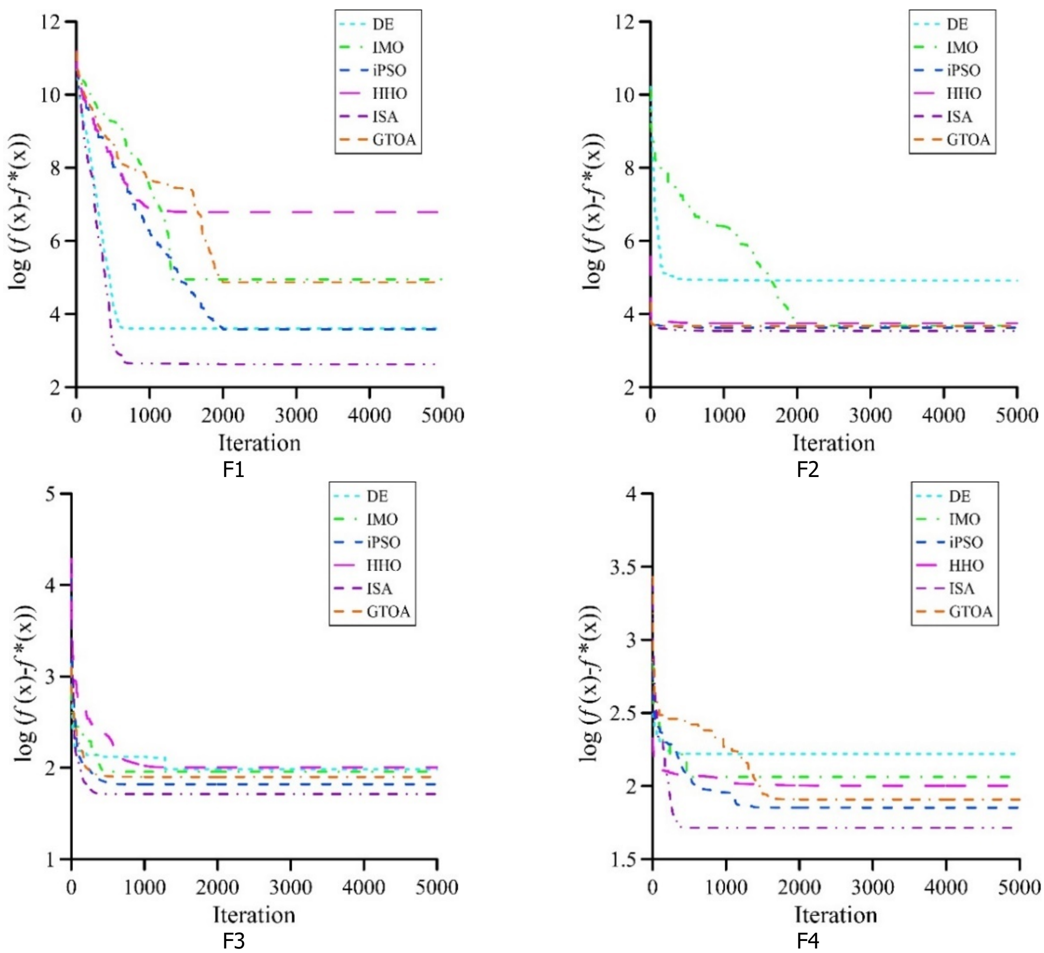
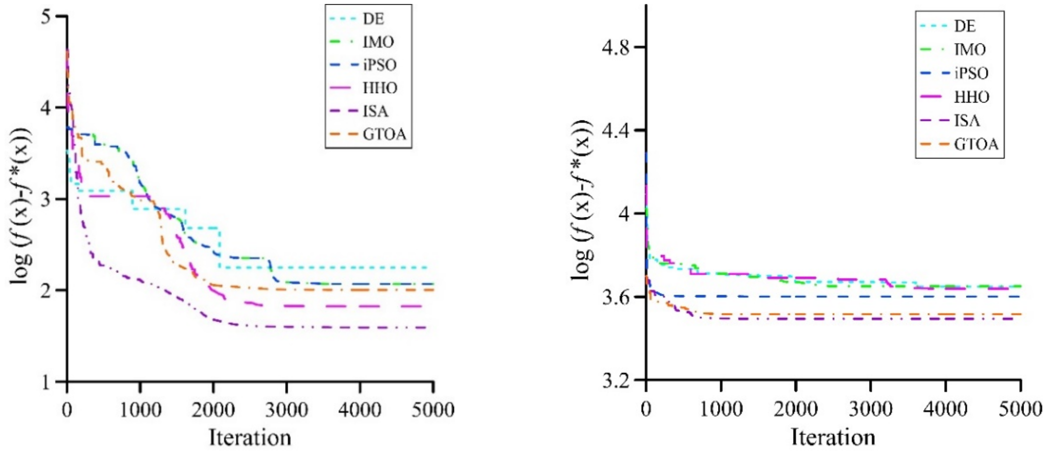


Fig.2. Convergence rate diagrams for selected functions

Fig 2. Cont'd



F5 F6

Table 10. Complexity analysis result for selected algorithms for D=30 (F6)

Algorithm	T_0	T_1	$\hat{T}_2$	$(\hat{T}_2-T_1)/T_0$	Rank
DE	1.40E-01	1.91E-01	4.54E+02	3.24E+03	1
IMO	1.40E-01	1.91E-01	6.40E+02	4.57E+03	6
iPSO	1.40E-01	1.91E-01	4.59E+02	3.28E+03	3
HHO	1.40E-01	1.91E-01	5.08E+02	3.63E+03	4
ISA	1.40E-01	1.91E-01	4.58E+02	3.27E+03	2
GTOA	1.40E-01	1.91E-01	6.05E+02	4.32E+03	5

Table 11. Complexity analysis result for selected algorithms for D=50 (F6)

Algorithm	T_0	T_1	$\hat{T}_2$	$(\hat{T}_2-T_1)/T_0$	Rank
DE	1.40E-01	3.06E-01	5.11E+02	3.65E+03	2
IMO	1.40E-01	3.06E-01	6.71E+02	4.79E+03	6
iPSO	1.40E-01	3.06E-01	5.19E+02	3.70E+03	3
HHO	1.40E-01	3.06E-01	6.44E+02	4.60E+03	4
ISA	1.40E-01	3.06E-01	5.07E+02	3.62E+03	1
GTOA	1.40E-01	3.06E-01	6.60E+02	4.71E+03	5

4.1.4. Non-parametric statistical tests for unconstrained mathematical functions

In the current section, to give an exhaustive view of the performance of the selected methods, a non-parametric statistical test (Friedman rank test) is implemented over the mean and standard deviation values. The achieved results are given in Table 12. Presented results indicate that ISA outperforms other methods in terms of stability and accuracy.

4.2. Case studies for thermo-economic optimization

In the current section, the mentioned six well-established optimization methods are employed for thermo-economic optimization for residential buildings. For this aim, five different cities from the Aegean geographic location of Turkey, as shown in Fig. 3, are selected.

Table 12. The Friedman rank test for mean and std values for unconstrained functions

Method	Test for the optimal mean value			Test for optimal Std. value		
	Friedman value	Normalized value	Rank	Friedman value	Normalized value	Rank
DE	22	0.181818	6	19	0.210526	4.5
IMO	20	0.200000	5	19	0.210526	4.5
iPSO	9	0.444444	2	8	0.5	2
HHO	17	0.235294	4	21	0.190476	6
ISA	4	1.000000	1	4	1	1
GTOA	12	0.333333	3	13	0.307692	3

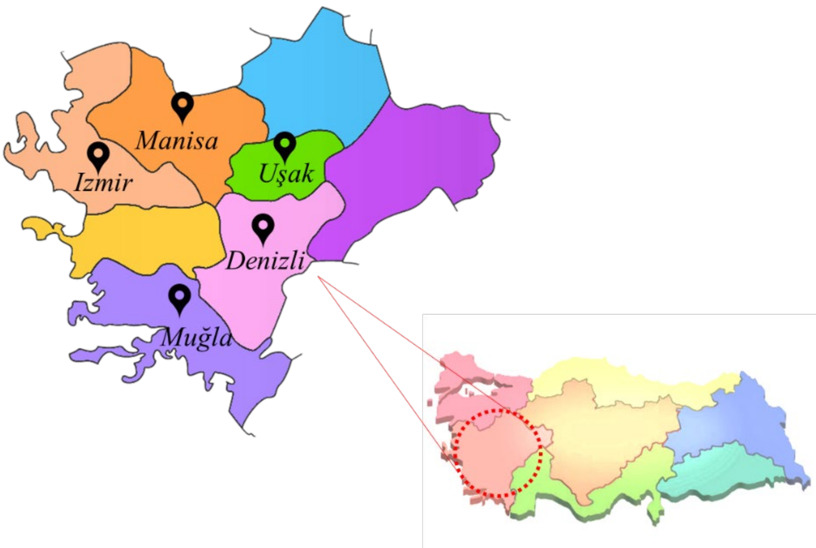


Fig. 3. Selected locations from Turkey

The heating degree-day values of these locations are considered in the thermal modeling of the system. The total heating cost of the insulated building is considered as the objective function of the optimization problem. The design variables of the optimization problem are from both continuous and discrete variable sets, which increases the complexity of the search space. The design variables, as presented in Table 13, are the fuel type, insulation material type and the thickness of the insulation layer. It should be mentioned that the monetary properties of the fuels and insulation material are taken from the government data series of 2016 [32]. In addition, the optimization process is run with 20 particles in 50 iterations (i.e. 1000 Objective Function Evaluations (OFEs)) for all optimization methods in all cases.

The optimal results obtained using different optimization techniques are comparatively presented in Table 14. To demonstrate the performance of the methods in solving the current optimization problem the convergence rate diagrams of the optimization process are given in Fig. 4. It is observed from the achieved results that the population-based optimization methods provide acceptable performance in solving the current problem for all cases. As seen from the convergence diagrams the ISA technique is more rapid than other techniques.

Table 13. Design variables of the optimization problem

Design variable	Var. type	No.	Type	Properties		
				Hu	η	C_{fuel}
Fuel type X_1	Discrete	1	Natural gas	34485000 (J/m^3)	0.90	0.385 (\$/m^3)
		2	Coal	25080000 (J/kg)	0.65	0.273 (\$/kg)
		3	Fuel-oil	40546000 (J/kg)	0.80	0.766 (\$/kg)
		4	LPG	45980000 (J/kg)	0.88	1.921 (\$/kg)
		5	Diesel	42911104 (J/kg)	0.84	1.614 (\$/kg)
				k (W/mK)	C_m (\$/m^3)	
Insulation material X_2	Discrete	1	Extruded polystyrene (XPS)	0.031	180	
		2	Expanded polystyrene (EPS)	0.039	120	
		3	Glass wool	0.040	75	
		4	Rock wool	0.040	80	
		5	Polyurethane	0.024	260	
				Interval (m)		
Insulation thickness X_3	Continuous	---	---	[0.0001,1.0]		

Table 14. Optimal results obtained by different optimization methods

Me	Optimal results	Uşak HDD=2414	Muğla HDD=1879	Denizli HDD=1627	Manisa HDD=1535	İzmir HDD=1118
DE	Insulation material	Glass wool	Glass wool	Glass wool	Glass wool	Glass wool
	Fuel type	Natural gas	Natural gas	Natural gas	Natural gas	Natural gas
	Opt. ins. thickness (m)	0.0963	0.0826	0.0755	0.0728	0.0591
	THC (\$/m2)	15.9608	13.9038	12.8331	12.4217	10.3798
IMO	Insulation material	Glass wool	Glass wool	Rock wool	Glass wool	Rock wool
	Fuel type	Natural gas	Natural gas	Natural gas	Natural gas	Natural gas
	Opt. ins. thickness (m)	0.1062	0.0726	0.0739	0.0924	0.1035
	THC (\$/m^2)	16.0335	13.9846	13.2046	12.6787	12.0904
iPSO	Insulation material	Glass wool	Glass wool	Glass wool	Glass wool	Glass wool
	Fuel type	Natural gas	Natural gas	Natural gas	Natural gas	Natural gas
	Opt. ins. thickness (m)	0.0963	0.0826	0.0755	0.0728	0.0591
	THC (\$/m^2)	15.9608	13.9038	12.8331	12.4217	10.3798
HHO	Insulation material	Glass wool	Glass wool	Glass wool	Glass wool	Glass wool
	Fuel type	Natural gas	Natural gas	Natural gas	Natural gas	Natural gas
	Opt. ins. thickness (m)	0.1082	0.0671	0.0744	0.0601	0.0758
	THC (\$/m^2)	16.0435	14.1117	12.8340	12.5716	10.5980
ISA	Insulation material	Glass wool	Glass wool	Glass wool	Glass wool	Glass wool
	Fuel type	Natural gas	Natural gas	Natural gas	Natural gas	Natural gas
	Opt. ins. thickness (m)	0.0963	0.0826	0.0755	0.0728	0.0591
	THC (\$/m^2)	15.9608	13.9038	12.8331	12.4217	10.3798
GTOA	Insulation material	Glass wool	Glass wool	Glass wool	Glass wool	Glass wool
	Fuel type	Natural gas	Natural gas	Natural gas	Natural gas	Natural gas
	Opt. ins. thickness (m)	0.0963	0.0826	0.0755	0.0728	0.0591
	THC (\$/m^2)	15.9608	13.9038	12.8331	12.4217	10.3798

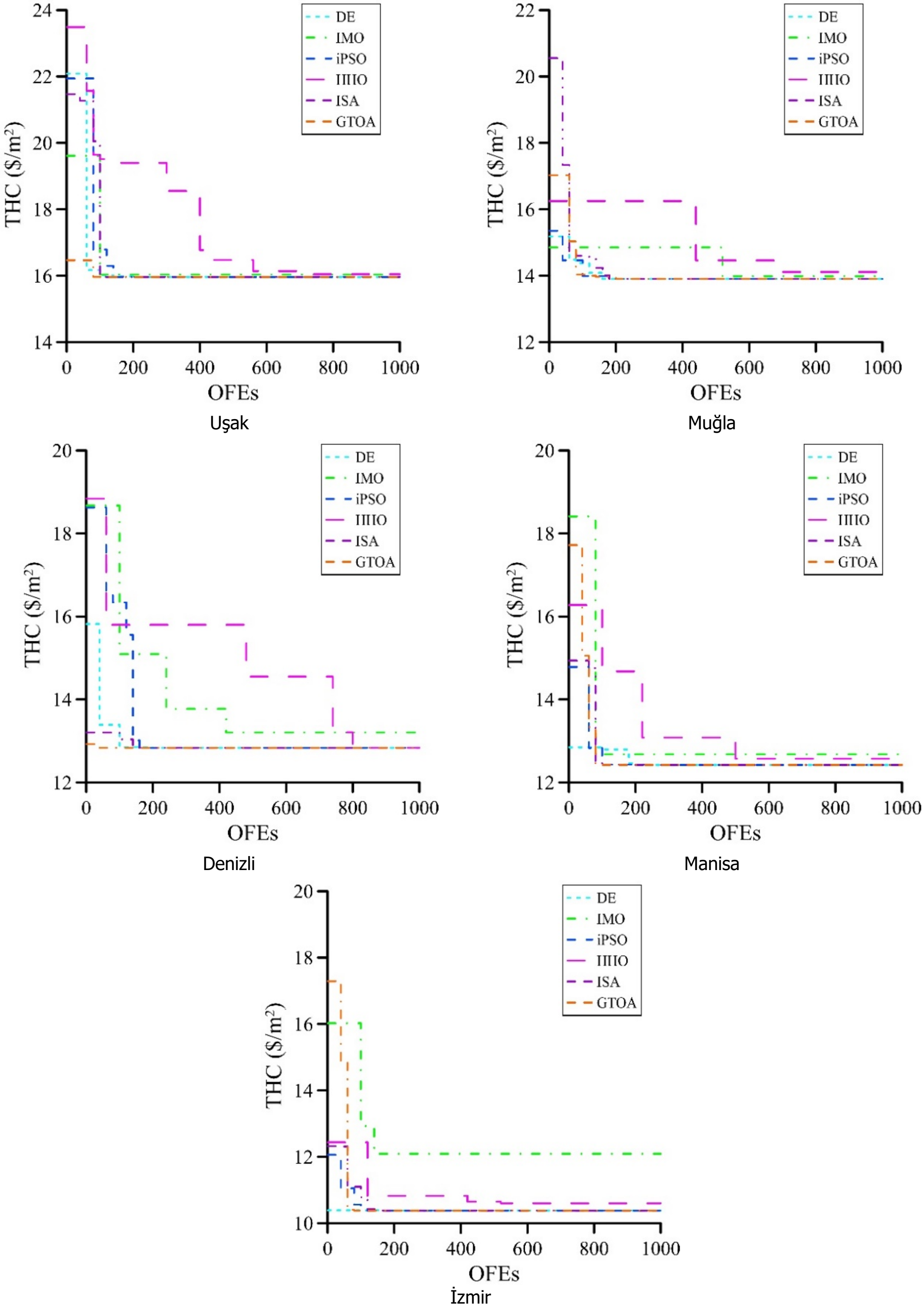


Fig 4. Convergence rate for different methods during the optimization process

5. Conclusion

In the current study, the performance of the population-based and non-gradient optimization techniques in solving the thermo-economic optimization model is investigated. For this aim, Differential Evolution (DE), Ions Motion Optimization (IMO), Integrated Particle Swarm Optimization (iPSO), Harris Hawks Optimization (HHO), Interactive Search Algorithm (ISA), and Group Teaching Optimization Algorithm (GTOA) are selected. Notably, testing the performance of the population-based techniques in solving the thermo-economic optimization model clarifies the effective feature of these methods and their applicability in different engineering optimization problems. In addition, to give a wide aspect for the researchers, the performance of the selected methods is also assessed in solving mathematical function problems with different properties. Distinct analyses such as accuracy and stability, convergence behavior, complexity, and non-parametric statistical tests are implemented on the obtained optimal results. According to the achieved outcomes, the performance of the ISA method due to its search paradigms outperforms the other techniques. The performance of ISA method is followed by the iPSO and DE methods.

Also, a thermo-economic optimization model is carried out for the exterior wall of the residential buildings. The total heating cost of the insulated building is minimized as the objective function of the model. The optimal values for the design variables (i.e. fuel type, insulation material, and insulation thickness) are obtained using the selected optimization methods. It should be noted that the inflation rate is assumed to represent the rate of increase in fuel prices. However, the rate of increase in fuel prices could be slightly different from the inflation rate. In the study cases, five different cities in the Aegean region of Turkey are considered. All selected techniques put forward an acceptable performance in solving the proposed thermo-economic optimization model. Particularly, ISA, iPSO, and DE present superior performance in comparison with other techniques in terms of accuracy and convergence rate. The optimal results for the selected case studies mostly are determined glass wool and natural gas as the insulation material type and fuel type, respectively. In the future, the performance of the different optimization methods would be tested on more real-world optimization problems and the design of zero-energy buildings.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest concerning the research, authorship, and/or publication of this article.

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